

The University of Chicago

A COMPARISON OF THE PHOTOMETRIC FIELDS OF THE 6-INCH
DOUBLET, 24-INCH REFLECTOR, AND 40-INCH
REFRACTOR OF THE YERKES
OBSERVATORY

WITH SOME INVESTIGATION OF THE ASTROMETRIC
FIELD OF THE REFLECTOR

A DISSERTATION
SUBMITTED TO THE FACULTY
OF THE OGDEN GRADUATE SCHOOL OF SCIENCE
IN CANDIDACY FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

DEPARTMENT OF ASTRONOMY AND ASTROPHYSICS

BY
ALICE HALL FARNSWORTH

Reprinted from
Publications of the YERKES OBSERVATORY, Vol. IV, Part V

THE UNIVERSITY OF CHICAGO PRESS
CHICAGO, ILLINOIS

1926

THE UNIVERSITY OF CHICAGO PRESS
CHICAGO, ILLINOIS

THE BAKER & TAYLOR COMPANY
NEW YORK

THE MACMILLAN COMPANY OF CANADA, LIMITED
TORONTO

THE CAMBRIDGE UNIVERSITY PRESS
LONDON

THE MARUZEN-KABUSHIKI-KAISHA
TOKYO, OSAKA, KYOTO, FUKUOKA, SENDAI

THE COMMERCIAL PRESS, LIMITED
SHANGHAI

The University of Chicago

A COMPARISON OF THE PHOTOMETRIC FIELDS OF THE 6-INCH
DOUBLET, 24-INCH REFLECTOR, AND 40-INCH
REFRACTOR OF THE YERKES
OBSERVATORY

WITH SOME INVESTIGATION OF THE ASTROMETRIC
FIELD OF THE REFLECTOR

A DISSERTATION
SUBMITTED TO THE FACULTY
OF THE OGDEN GRADUATE SCHOOL OF SCIENCE
IN CANDIDACY FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY
DEPARTMENT OF ASTRONOMY AND ASTROPHYSICS

BY
ALICE HALL FARNSWORTH

Reprinted from
Publications of the YERKES OBSERVATORY, Vol. IV, Part V

THE UNIVERSITY OF CHICAGO PRESS
CHICAGO, ILLINOIS

1926

THE UNIVERSITY OF CHICAGO PRESS
CHICAGO, ILLINOIS

THE BAKER & TAYLOR COMPANY
NEW YORK

THE MACMILLAN COMPANY OF CANADA, LIMITED
TORONTO

THE CAMBRIDGE UNIVERSITY PRESS
LONDON

THE MARUZEN-KABUSHIKI-KAISHA
TOKYO, OSAKA, KYOTO, FUKUOKA, SENDAI

THE COMMERCIAL PRESS, LIMITED
SHANGHAI

The University of Chicago

A COMPARISON OF THE PHOTOMETRIC FIELDS OF THE 6-INCH
DOUBLET, 24-INCH REFLECTOR, AND 40-INCH
REFRACTOR OF THE YERKES
OBSERVATORY

WITH SOME INVESTIGATION OF THE ASTROMETRIC
FIELD OF THE REFLECTOR

A DISSERTATION
SUBMITTED TO THE FACULTY
OF THE OGDEN GRADUATE SCHOOL OF SCIENCE
IN CANDIDACY FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY
DEPARTMENT OF ASTRONOMY AND ASTROPHYSICS

BY
ALICE HALL FARNSWORTH

Reprinted from
Publications of the YERKES OBSERVATORY, Vol. IV, Part V

THE UNIVERSITY OF CHICAGO PRESS
CHICAGO, ILLINOIS

1926



Digitized by the Internet Archive
in 2026

https://archive.org/details/IA41533907_0091

A COMPARISON OF THE PHOTOMETRIC FIELDS OF THE 6-INCH DOUBLET, 24-INCH REFLECTOR, AND 40-INCH REFRACTOR OF THE YERKES OBSERVATORY, WITH SOME INVESTIGATION OF THE ASTROMETRIC FIELD OF THE REFLECTOR¹

BY ALICE HALL FARNSWORTH

I. COMPARISON OF THE PHOTOMETRIC FIELDS OF THE 6-INCH DOUBLET, 24-INCH REFLECTOR, AND 40-INCH REFRACTOR

INTRODUCTION

The first part of this paper comprises (1) a comparison of the behavior of stellar images on photographic plates taken with the 6-inch doublet, 24-inch reflector, and 40-inch refractor, in its relation to the determination of photographic and photovisual magnitudes, together with the characteristics of the optical system which produces the images and (2) a study of the results obtained, much of the data being furnished by plates on the twenty-four Kapteyn areas at declination $+45^\circ$. The first involves a study of the calibration of the field with the preparation of tables of corrections to the center, of the color-curves, of the curves of zonal foci as shown by the Hartmann test, and of the behavior of the parallel-wire grating as applied for the determination of scale. The second includes a comparison of the actual sizes of images used, correlated with precision of the photometric results, and a comparison of the efficiency of the three instruments as shown by curves connecting exposure-time with limiting magnitude.

INSTRUMENTS, PLATES, AND MEASUREMENT

A brief description will first be given of the telescopes, plates, and mode of measuring. Table I gives, for the three telescopes, the usual designation of the instrument, its type together with the part of the spectrum for which the optical system is corrected, its form, the full aperture in inches and millimeters with the aperture most used in actual practice, the focal length in inches and

TABLE I

DESIG.	TYPE		FORM	APERTURE			FOCAL LENGTH		SCALE 1 mm =
				Full		Used			
UV	Refractor	Photog.	Petzval doublet	in. 5.7	mm 145	in. 4	in. 32	mm 814	253"
R.	Reflector	Achrom.	Newtonian	23.5	597	12, 18	93	2360	87.4
40-inch	Refractor	Visual	Clark	40	1016	40	744	19340	10.66

millimeters, and the approximate scale of the plate. The plates taken with the three telescopes constitute the UV, R, and O series, respectively. The designation of the 6-inch comes from the fact that the lenses and objective prisms used on this telescope are made from the Jena ultra-violet flint glass. An idea of its transparency is given from the statements found in *Physikalische Zeitschrift*, 4, 751, 1903, that at λ 3050, 50 per cent of the original intensity goes through 1 cm of

¹ A dissertation submitted to the Faculty of the Ogden Graduate School of Science of the University of Chicago in candidacy for the degree of Doctor of Philosophy, 1920.

glass, at λ 2800, 50 per cent of the original goes through 1 mm of glass. The average thickness of the front pair of lenses of the doublet is 26 mm, of the rear pair 21 mm, and of the 15° prism 26 mm.

Photographic magnitudes have been largely determined from Seed 30 plates (sensitive from λ 3500 to λ 5000, maximum about λ 4200), photovisual from Cramer Isochromatic Instantaneous and Medium plates (sensitive, in addition, between λ 5000 and λ 5800) always placed behind a yellow filter. The terms "Seed" and "Cramer" used for brevity in what follows are thus to be

TABLE II
FILTERS

SERIES	FILTER	SIZE IN INCHES	KIND	MAKER	TRANSMISSION	USED	
						From	To
UV.....	Beta 10	4×5	Gelatine	Wallace	5000-6000	1907 Feb. 20	1924
R.....	Beta 7	3×3	Gelatine	Wallace	4800-6000	1906 July 24	1914 Sept. 30
R.....	W 12	3×3	Gelatine	Wratten	5000-6000	1914 Sept. 30	1917 July 9
R.....	J1	3×3	Colored glass	Schott	4800-5900	1917 July 9	1924
O.....	Dec. '07	8×10	Gelatine	Wallace	4900-6000	1907 Dec. 15	1920 June 2
O.....	M 1	8×10	Gelatine	Mees	4900-6000	1920 June 5	1924

understood as synonymous with photographic and photovisual work, respectively. Table II gives a description of the filters used, with the region of wave-length which each transmits. The transmission was determined from exposures of Cramer Isochromatic plates through the filter to the northern sky in a Wallace spectrograph. The test of β 10 only was made on a Cramer Trichromatic plate, which shows a maximum sensitiveness somewhat nearer the blue than the Isochromatic, but a test of "Dec. '07" on the same plate shows that the two filters are very much alike. All consist of a gelatine film between plates of optical glass (β 10 only uses ordinary commercial glass), except J 1 which is made of tinted Jena glass.

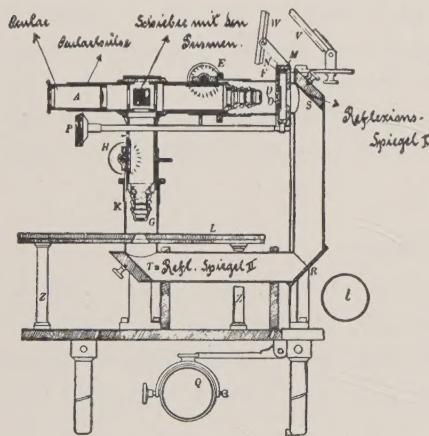


FIG. 1.—Diagram of Hartmann Microphotometer.

The plates are measured in the Hartmann Microphotometer,¹ a diagram of which is shown in Figure 1. Light from an electric lamp l passes through ground glass at R , through tubes to be reflected by mirrors T and S into tubes at right angles to the first pair. L is a table upon which rests the photographic plate to be measured. At the aperture in this table the light reflected from T passes through the plate upon a microscope objective G , whence it falls upon a Lummer-Brodhun cube. The lower half of the diagonal face separating the two half-prisms which compose the cube is silvered, so that light falling upon it is reflected into the eyepiece A . Thus when a star-image is brought under the microscope at G , it appears in the lower half of the field of view of the eyepiece at A . N indicates the position of a frame for holding the artificial scale with which the plate-images are compared. In this frame the scale is mounted so that the row of images comes just above the middle of the aperture at O . Light reflected from the mirror S illuminates the scale and passes through the microscope-objective D to the cube. The lower silvered face blocks its passage so that it passes only through the upper half of the cube, and the scale-row appears in the upper half of the eyepiece A . The frame holding the scale can readily be moved in the direction of its length by means of a rack and pinion P . Focusing screws at H and E make it possible to bring both sections of the field into sharp focus, when the dividing line in the middle practically disappears.

¹ *Astrophysical Journal*, 10, 321, 1899; 36, 171, 1912.

The lamp l has been mounted upon a sliding frame so that the illumination of the upper and lower parts of the field may be made alike. This adjustment is necessary because of the varying density of the film for the various plates used. The image I to be measured is brought approximately to the center of the field, just below the dividing line. The scale-row is moved along until an image a brighter than I appears at its left, and an image b fainter than I at its right. The interval aI is then estimated in tenths of the interval ab .

The artificial scale referred to is a row of images made by successive exposures on a star, the times of exposure being such that their logarithms increase uniformly. Scales made under varying conditions are useful for plates of different quality. The scales used for the present series of plates are listed in Table XXIII (page 27) and discussed in a later paragraph. In certain cases where irregularities in the scale-row were noted, measures of diameters or comparison with tested scales made it possible to determine corrections to be applied to the images affected. In connection with Scales UV 5(442) and R 16, a supplementary row formed by a fainter star close by was found useful for measuring the faintest images. The series of measures, after being corrected for irregularities of scale, is ready for the correction for distance from the center, which depends on the

CALIBRATION OF THE FIELD

24-INCH REFLECTOR

The general method for the calibration of the field, with the determination of corrections to reduce all images to the center of the plate, was the same for the three instruments, though each

TABLE III
REFLECTOR PLATES FOR CALIBRATION

CRAMER ISO, FILTERS $\beta 7$, J1. SCALE R 5					SEED 30. SCALE R 16				
R No.	Exp. Time	No. of Exp.	No. of Rows		R No.	Exp. Time	No. of Exp.	No. of Rows	
			Meas.	Used				Meas.	Used
Aperture 18 in.					Aperture 18 in.				
2596.....	30 ^s	15	3	0	2595.....	4 ^s	12	1	1
2751.....	5, 8, 10 ^s	13	3	0	2762.....	10, 15, 20 ^s	12	3	1
2754.....	15, 20, 25, 30 ^s	10	4	0	2766.....	15 ^s	11	3	2
3630.....	30, 60 ^s	14	4	4	3629.....	10, 25 ^s	11	4	3
3634.....	10, 20 ^s	16	3	1	3639.....	1 ^m	9	1	1
3637.....	3 ^m	14	4	1	3658.....	10, 25, 50 ^s	17	6	6
3638.....	5 ^m	9	3	2	3662.....	3, 6 ^s	17	2	0
3642.....	5, 7 ^s	22	2	0					
3643.....	4, 8, 16 ^s	29	3	1					
3647.....	3, 6, 4, 5 ^s	29	7	1					
3648.....	10, 20, 30 ^s	19	3	1					
3653.....	2, 4, 8 ^s	21	3	1					
3655.....	5, 10, 20 ^s	13	1	1					
3656.....	8, 10, 12 ^s	9	3	2					
Sums			46	15				20	14
Aperture 12 in.					Aperture 12 in.				
3653.....	2, 4, 8 ^s	21	2	2	3657.....	5, 10, 20 ^s	11	8	6
3663.....	5, 10, 30, 60 ^s	25	6	6	3662.....	4, 8 ^s	17	2	0
Sums			8	8	3664.....	2, 4, 8 ^s	20	6	6
								16	12

offered its special problems in working out the details. The field of the reflector was perhaps the simplest and will be described first.

The plates and their measurement.—Practically all the plates were centered at Polaris or at North Polar Sequence No. 5, which with B.D. $+88^{\circ}5$ makes a group with a convenient range of magnitude. Exposures of equal length so arranged that they extend across the measurable field, the slow motion screw of the telescope being turned between each two exposures, furnish a series of images of each star at varying distances from the optical axis. The exposures were so short that no trouble with uniformity of seeing during a given series occurred and enough exposures were made to average out inaccuracies in the timing of the shortest exposures. The images were measured in rows, usually

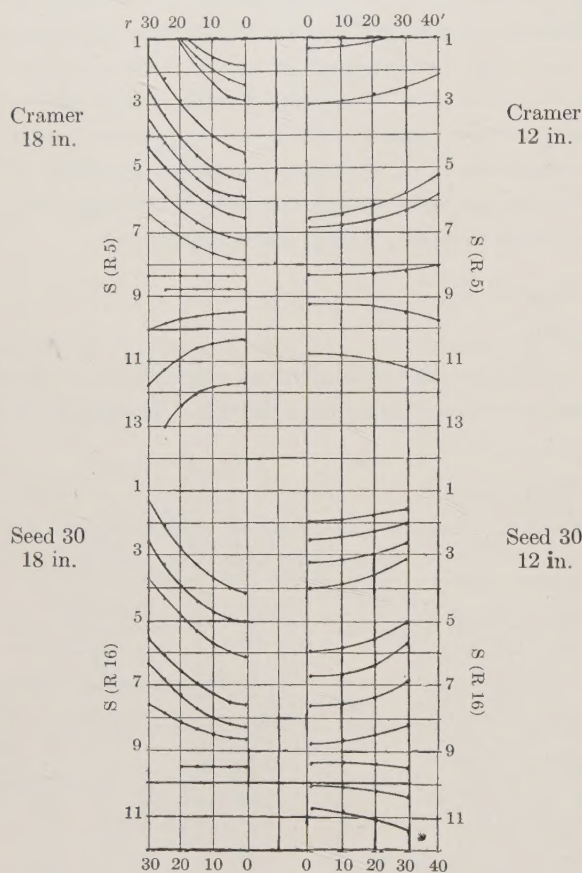


FIG. 2.—Preliminary curves. Reflector

twice, by artificial scales: R 16 for the Seed plates, R 5 for the Cramer. Table III gives data for the four groups of plates measured. The kind of plate, aperture, and scale used for measurement are given in the headings. The R number of the plate, the exposure times, and the number of measurable images in a row are given in the first three columns. The fourth column gives the number of rows actually measured on each plate, and the last, the number of these rows selected to form part of the final set of curves.

Preliminary curves.—With the mean scale-reading of each image was tabulated its distance from the optical axis. These distances on the reflector plates are determined with the help of a glass plate carrying concentric circles and lines radiating from a center, the position of the optical axis on the plate being estimated by means of the radially distorted images in the outer parts of the plates. A curve, whose co-ordinates were the measured scale-reading (S) and distance from axis (r), was plotted for each row of images. From the 90 smooth curves so made, Figure 2 shows the group of 49 representative and well-distributed curves chosen to form the basis for the correction-tables. Only one branch of each curve is shown, as the curves are symmetrical with respect to the center.

Curves for images fainter than $S=9.5$ are lacking on the 18-inch Seed plates. Provisional values were inserted in the table, based on the ratios between the corrections for 12- and 18-inch apertures on the Cramer plates in the corresponding regions. These will be improved by comparisons of magnitudes from 12- and 18-inch exposures on some of the Kapteyn fields.

The final tables based on the four sets of curves shown in Figure 2 are designated R(C18), R(C12), R(S18), R(S12), the parentheses denoting kind of plate and aperture. The method of obtaining the correction-table from the preliminary curves is the same for all four. The work will be given for Table R(C18) only.

Interpolation curves.—From each of the preliminary curves was read the scale-reading at $r=0'$, $5'$, $10'$, $15'$, $20'$, $25'$. These are shown in Table IV, where for each scale-reading not at the center is given the correction necessary to reduce it to the scale-reading for $r=0$. Figure 3 shows the interpolation curves, plotted with measured S and correction to measured S , as co-ordinates, for each value of r for which tabulation is made in Table IV.

Intermediate curves.—A further smoothing-out of the material was made by plotting the intermediate curves shown in Figure 4. A curve is drawn for each half unit of scale-reading, the coordinates r and correction to S being read directly from the interpolation curves in Figure 3.

TABLE IV
INTERPOLATION CURVES

S at $r=0'$	S at $r=5'$	Correction to Meas. S	S at $r=10'$	Correction	S at $r=15'$	Correction	S at $r=20'$	Correction	S at $r=25'$	Correction
2.93.....	2.76	+0.17	2.38	+0.55	1.71	+1.22	0.70	+2.23		
4.50.....	4.32	+ .18	3.97	+ .53	3.49	+1.01	2.85	+1.65	2.20	+2.30
5.33.....	5.22	+ .11	4.99	+ .34	4.54	+0.79	3.98	+1.35	3.31	+2.02
5.90.....	5.80	+ .10	5.63	+ .27	5.23	+ .67	4.72	+1.18	4.14	+1.76
6.45.....	6.38	+ .07	6.15	+ .30	5.81	+ .64	5.38	+1.07	4.85	+1.60
7.20.....	7.10	+ .10	6.90	+ .30	6.62	+ .58	6.27	+0.93	5.83	+1.37
7.80.....	7.74	+ .06	7.60	+ .20	7.38	+ .42	7.08	+ .72	6.75	+1.05
8.60.....	8.60	.00	8.60	.00	8.60	.00	8.60	.00	8.60	0.00
9.41.....	9.43	— .02	9.49	— .08	9.53	— .12	9.64	— .23	9.80	— .39
10.28.....	10.30	— .02	10.38	— .10	10.54	— .26	10.84	— .56	11.22	— .94
11.65.....	11.67	—0.02	11.75	—0.10	11.96	—0.31	12.36	—0.71	13.00	—1.35

Correction table.—The final table R(C18) shows the correction to the measured S read from Figure 4 for each half unit of scale-reading, for each minute of arc from $r=1'$ to $24'$. Table V reproduces Table R(C18) in condensed form.

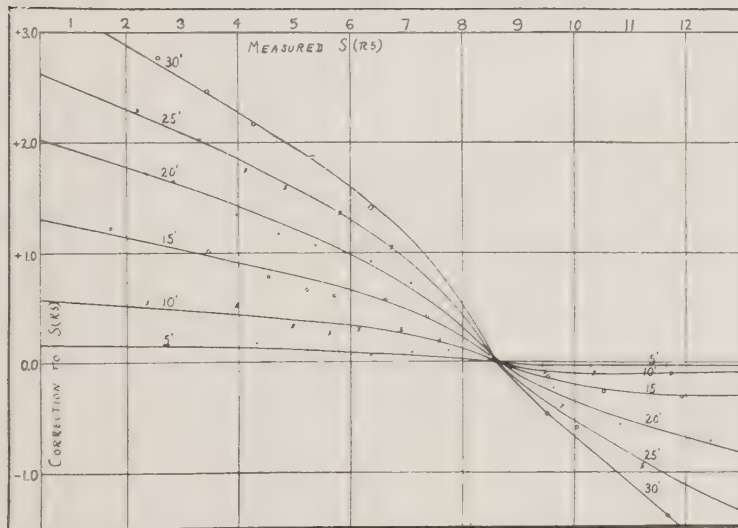


FIG. 3.—Interpolation curves

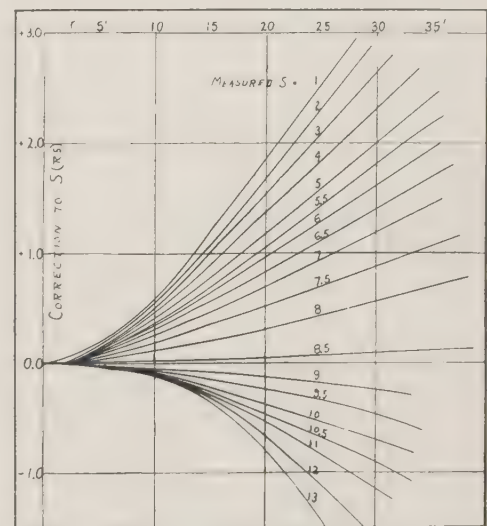


FIG. 4.—Intermediate curves

Reflector, 18-inch aperture; Cramer Iso with filter J 1

40-INCH REFRACTOR

The problem for the 40-inch telescope presents especial difficulties because it is almost impossible to find a night when the seeing remains so uniform that a series of successive equal exposures can be depended upon to give images of uniform size on the plate. Incidentally the same trouble enters in obtaining artificial scales of smooth gradation and varying quality to match plates taken under varying atmospheric conditions. The data used for the final table were obtained from four plates concerning which Table VI gives the necessary facts. Each row of images extends about half-way across the plate.

The plotted points in each separate row, using scale-reading and distance from center (r) as co-ordinates, gave evidence of irregularities in seeing so great as completely to mask any variation of size of image with varying r . The first step, therefore, was a determination of corrections for irregularities of exposure.

The stars were divided into groups according to size of image on the plate, each group including images of nearly the same size so as to be similarly affected, presumably, by atmospheric changes.

TABLE V
R PHOTOVISUAL CORRECTIONS
(For 18-in. aperture, expressed in terms of scale-readings)

r S_{RS}	2'	4'	6'	8'	10'	12'	14'	16'	18'	20'	22'	24'
1.0.....	+0.06	+0.15	+0.26	+0.42	+0.60	+0.83	+1.07	+1.34	+1.62	+1.88	+2.15	+2.41
2.0.....	+ .05	+ .13	+ .24	+ .38	+ .54	+ .75	+0.97	+1.21	+1.45	+1.70	+1.95	+2.20
3.0.....	+ .05	+ .11	+ .22	+ .35	+ .51	+ .70	+ .91	+1.13	+1.34	+1.55	+1.77	+1.99
4.0.....	+ .04	+ .11	+ .20	+ .32	+ .45	+ .63	+ .80	+1.00	+1.18	+1.38	+1.58	+1.77
5.0.....	+ .03	+ .09	+ .17	+ .28	+ .41	+ .55	+ .70	+0.85	+1.02	+1.18	+1.36	+1.51
6.0.....	+ .03	+ .08	+ .15	+ .23	+ .34	+ .46	+ .58	+ .71	+0.84	+0.97	+1.10	+1.23
7.0.....	+ .02	+ .06	+ .12	+ .18	+ .26	+ .35	+ .43	+ .52	+ .60	+ .70	+0.80	+0.90
8.0.....	+ .02	+ .03	+ .06	+ .08	+ .12	+ .15	+ .18	+ .22	+ .27	+ .32	+ .37	+ .42
9.0.....	.00	- .01	- .02	- .03	- .04	- .05	- .06	- .08	- .09	- .12	- .14	- .17
10.0.....	.00	- .02	- .03	- .07	- .10	- .14	- .19	- .25	- .30	- .36	- .43	- .50
11.0.....	.00	- .02	- .03	- .07	- .10	- .17	- .24	- .33	- .42	- .53	- .65	-0.77
12.0.....	0.00	-0.02	-0.03	-0.07	-0.10	-0.17	-0.26	-0.37	-0.50	-0.65	-0.83	-1.00

From each group were selected rows of images so placed on the plate that the average r of the images on the preceding side was practically equal to the average r of those on the following side. For images thus symmetrically placed, the assumption was made that the average scale-reading (S) for each exposure should equal that for every other if the seeing were uniform. The mean of the average scale-readings for the separate exposures was adopted as standard in each set of stars, and the differ-

TABLE VI
40-INCH PLATES FOR CALIBRATION

O No.	Field	Exp. Time	No. Exp.	No. Rows Measured	O Scale Used	Quality of Plate
83.....	χ Persei	10 ^m	8	25	12	Fair to Good
103.....	Praesepe	3	18	19	12	Fair to Good
295.....	χ Persei	10	8	46	26	Excellent
296.....	χ Persei	10	8	30	19	Good

ence between this mean and the average for each exposure was adopted as a correction for irregularities of seeing, to be applied to every image on the plate obtained from the given exposure and included in the group. An idea of the size of these corrections to scale-reading, as well as their variation with the size of the image, may be gained from Table VII which gives for the four plates the corrections in scale-units for each exposure on each group whose mean scale-reading appears at the head of the column. (Note from Table XXIII that one scale-interval corresponds approximately to half a magnitude.) On plate O 83 the correction obtained from each group was applied to stars in that group, as there was a slight indication of progressive change in size of correction with change in size of image. Plate O 103 furnished one group only suitable for determining the correction, and the corrections from this group were applied to all the images on the plate. On plates O 295 and

O 296, no progressive change in correction being noted, the mean correction from all the groups was applied to all the images.

The corrections for irregularity of exposure having been applied, the next step was the derivation of mean curves on each plate. The fields selected offer such an abundance of material that it

TABLE VII
CORRECTIONS IN SCALE-UNITS FOR IRREGULARITIES OF EXPOSURE

PLATE O 83. SCALE O 12					PLATE O 296. SCALE O 19				
Mean S Exp. No.	5.85	7.92	9.68	11.31	5.74	6.68	7.79	8.51	Mean Correction
1.....	+0.4	+0.1	0.0	-0.2			-0.1	-0.1	-0.1
2.....	-.1	-.1	.0	+.1			.0	-.1	.0
3.....	-.1	-.1	.0	+.1		-0.5	-.6	-.6	-.6
4.....	-.1	-.1	-.1	.0	0.0	-.1		.0	.0
5.....	.0	.0	-.1	-.1	.0	+.1	-.1	.0	.0
6.....	+.1	.0	.0	.0	.0	+.1	+0.1	+.1	+.1
7.....	.0	+.1	.0	.0	.0	0.0		-.1	.0
8.....	+0.2	+0.1	+0.2	0.0	+0.1			+0.2	+0.1

PLATE O 295. SCALE O 25						PLATE O 103. SCALE O 12. Mean S 4.91				
Mean S Exp. No.	4.30	6.01	6.27	7.37	8.61	Mean Correction	Exp. No.	Correction	Exp. No.	Correction
1.....	-0.14	-0.07	-0.14	-0.26	-0.01	-0.12	1.....	0.0	10.....	-0.2
2.....	-.08	-.01	.00	+.01	.00	-.02	2.....	-.2	11.....	-.2
3.....	+.01	-.02	-.02	-.01	+.01	-.01	3.....	+.1	12.....	.0
4.....	-.08	-.14	-.09	+.01	-.03	-.07	4.....	+.1	13.....	.0
5.....	+.09	-.02	+.08	+.09	.00	+.05	5.....	.0	14.....	.0
6.....	-.63	-.60	-.61	-.56	-.44	-.57	6.....	+.3	15.....	-.2
7.....	+.20	+.10	+.23	+.18	-.03	+.14	7.....	+.1	16.....	-1.0
8.....	+0.21	+0.28	+0.19	+0.14	+0.01	+0.17	8.....	-.1	17.....	+0.2
							9.....	+0.4	18.....	

seemed wise to group images according to size, and plot the mean instead of separate values. For this purpose the mean S for the images of each star falling within 10' of the center was determined. Groups of stars (from 3 to 8 in number) nearly alike in size were chosen and an arbitrary correction applied to all the images of each row to reduce all the stars in each group to a mean S at the center.

TABLE VIII
PHOTOVISUAL CORRECTIONS FOR 40-INCH PLATES

r S_{25}	5'	10'	15'	20'	25'
2.00.....	+0.02	+0.09	+0.21	+0.41	+0.70
4.00.....	.00	+.06	+.19	+.36	+.60
6.00.....	.00	+.05	+.15	+.31	+.47
8.00.....	.00	+.03	+.10	+.16	+.23
10.00.....	.00	-.04	-.09	-.15	-.22
12.00.....	-0.10	-0.17	-0.30	-0.42	-0.50

Tabulation of these corrected values according to r made it possible to obtain well-determined mean values of r and S which were used as co-ordinates for plotting. The four plates furnished 20 of these mean curves, derived from 110 rows of images (Fig. 5).

Since the final table was to be expressed in terms of scale-readings on scale O 25, the mean curves derived from measures with other scales had to be translated by means of appropriate curves into terms of O 25. From the preliminary curves, the final table O 2(25) was obtained by the method previously given for the reflector. The table is shown in condensed form in Table VIII.

Later a revision of table O 2 was undertaken because of accumulating evidence that negative corrections to faint images far from the axis were too large. The magnitudes obtained from 115 spectral images on 15 different plates of Kapteyn areas were compared with final mean magnitudes from other sources. The comparison showed the magnitudes from spectral images decreasing numerically with increase of S beyond 10. The tabular corrections for S greater than 10 were revised accordingly and incorporated in tables O 3(25) and O 3(26).

6-INCH UV DOUBLET

The calibration of the field of the 6-inch telescope offers difficulties not met in the case of the other two telescopes, because the shape of the preliminary curves is a sensitive function (1) of the size of the image on the plate and (2) of the focal setting of the plate. The main problem consists in the selection of curves that shall be at once consistent and representative.

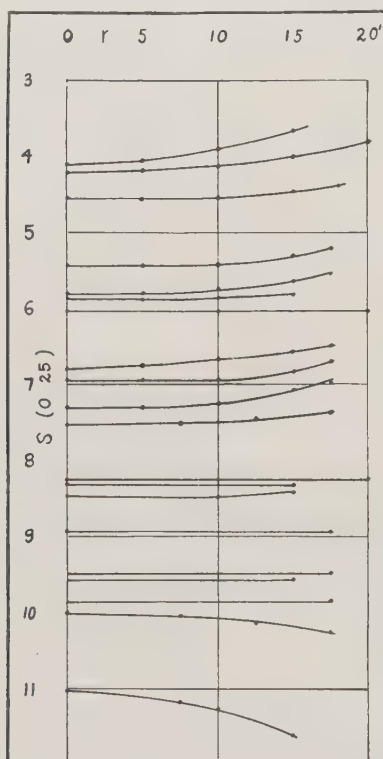


FIG. 5.—Preliminary curves. 40-inch refractor

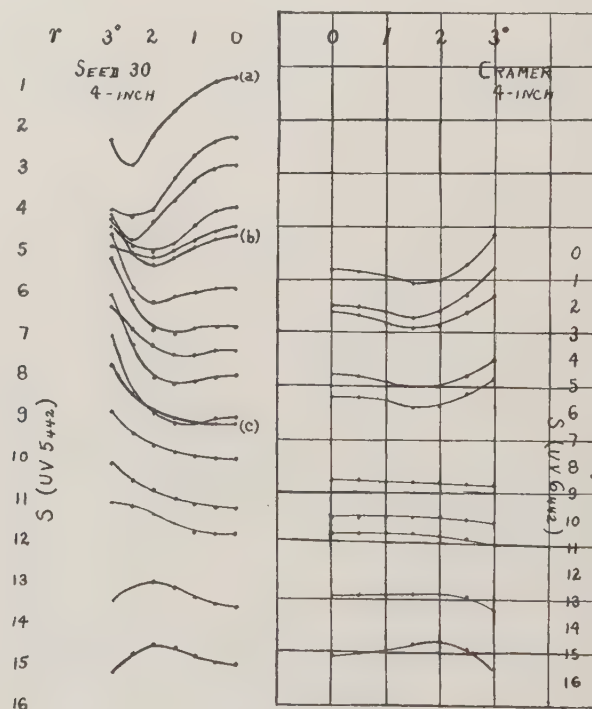


FIG. 6.—Preliminary curves. UV doublet

1. Comparison of the three curves (a), (b), (c) in Figure 6, which represent rows of images on the same plate (UV 1262) shows that for a given focal setting the shape of the preliminary curve depends decidedly on the size of the image. In the case of the 24-inch and 40-inch fields (compare Figs. 2 and 5) reduction of images to the center of the plate means reduction to the position of best focus, for images of all sizes. In the case of the 6-inch, only the smaller images have their best focus at the center of the plate. As the images increase in size, the position of best focus on the plate, i.e., of the smallest images in the calibration row, moves away from the optical axis.

2. Figure 7 shows for an image of given size ($S=6$) at its best focus the effect of changing the focal setting of the telescope. It is thus apparent that different plates may be expected to give calibration curves which are accordant only if they are taken at the same equivalent focal setting. Figure 7 shows that a change of 0.1 mm in the focal setting may change the size of the image on

the axis by 0.8 of a scale-unit. This change in focal setting corresponds to a change in temperature of only 4° centigrade, a change which often occurs in the course of an evening's work. A criterion of the fitness of a given plate, more accurate than the simple record of the temperature and focal setting used, is furnished on several of the plates by exposures with the grating R 6. This grating, specifications for which are given in Table XXII (page 24), has such a narrow "grating space" (see page 23) that the spectral images are of considerable length and show distinctly points of best focus at different wave-lengths, as the distance from the optical axis changes. That the "spectral distance," measured between the center of the central image and the point of best focus in the

spectrum, is also very directly a function of the focal setting is shown in Figure 8 where the results of plotting spectral distance against focal setting on two plates are shown. The spectral distance at the center of the plate increases (on upper branch of curve) by about 0.01 mm for an increase in focus of 0.03 mm.

Table IX shows the plates from which the preliminary curves of Figure 6 were selected. The data are the same as those appearing in Tables III and VI with the addition of the last

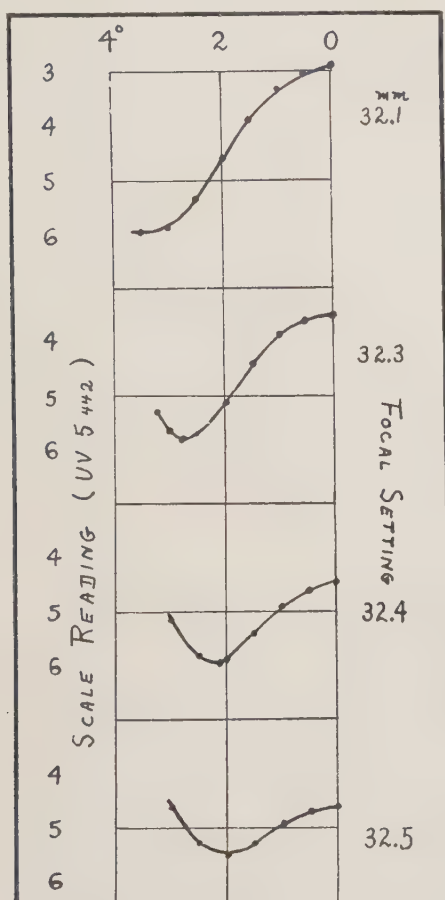


FIG. 7.—UV calibration. Focus plate UV 1484

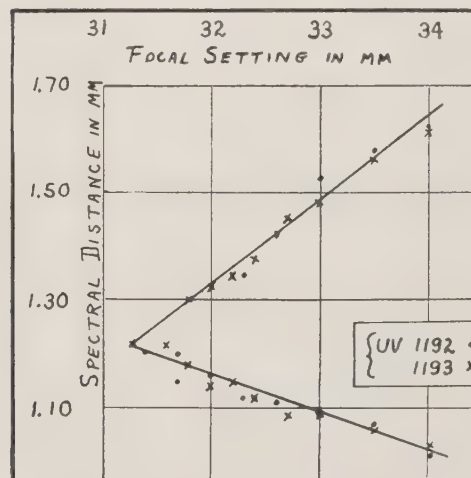


FIG. 8.—Spectral distances. Grating R 6

two columns giving the grating used and the spectral distance at the center. Of the five plates having exposures with R 6, UV 1696 shows a spectral distance at the center smaller than the other four, and the rise at the center for two of the curves used is correspondingly larger. The other plates gave curves which were consistent with the first five, as shown in Figure 6. All the plates were centered near Polaris, except UV 1260 and UV 1262 which carry exposures on the Pleiades.

The preliminary curves for Cramer Iso plates taken behind the yellow filter β 10 (this combination makes use of λ 5000–5800) are also given in Figure 6 and show in general the same characteristics as the curves on the Seed 30 plates (see page 22). The plates used, all centered near Polaris, are also shown in Table IX. They contain no exposures with R 6, but were selected for their general conformity to the character of the Seed 30 curves. The last column of Table IX shows the difference between the focal setting used and that indicated by the temperature-focus curve.

The correction tables $D(S)$ (for the Seed photographic plates) and $C(C)$ (for the Cramer photo-visual) were deduced from the preliminary curves in a manner similar to that previously described. Table X gives the first of these tables in condensed form.

The shape of the calibration curves in the case of each instrument is of course a function of the optical system used and depends on the aberrations—chromatic or spherical. It therefore seems best to leave the discussion of causes for the various forms until after an investigation (1) of the color-curves of the instruments and (2) of the performance of the various zones of the lens or mirror as shown by the Hartmann test.

TABLE IX
UV PLATES FOR CALIBRATION

SEED 27 AND 30. SCALE 5(442). 4-INCH							CRAMER ISO, BETA 10. SCALE 6(442). 4-INCH					
UV No.	Exp. Time	No. of Exp.	No. Rows		Grating	Spect. Dist.	UV No.	Exp. Time	No. of Exp.	No. Rows		Correc- tion to Focus
			Meas.	Used						Meas.	Used	
1164.....	m s	18	1	1	R 6	1.34	1256.....	s	14	1	1	0.00
1168.....	8	22	2	1	R 6	1.33	1263.....	14, 60	16	5	5	+ .01
1169.....	8	22	2	2	R 6	1.34	1358.....	29	13	5	1	- .04
1170.....	9	17	3	1	R 6	1.34	1365.....	19, 59	15	5	4	- .01
1214.....	30	18	2	2	R 7		1490.....	20, 10	15	1	1	+ .01
1260.....	9	16	6	1			1492.....	20	17	2	2	.00
1262.....	9	16	6	3			1494.....	10	16	2	1	.00
1267.....	19	18	2	1	R 8							
1693.....	5, 5	12	8	1								
1696.....	5, 19	13	5	4	R 6	1.31						

TABLE X
UV PHOTOGRAPHIC CORRECTIONS

$\begin{array}{c} r \\ S_6 \end{array}$	0°5	1°0	1°5	2°0	2°5	3°0
0.0.....	-0.14	-0.52	-0.99	-2.08		
2.0.....	- .13	- .42	- .94	-1.87		
4.0.....	- .11	- .31	- .81	-1.44	-1.87	
6.0.....	- .08	- .20	- .45	-0.46	+0.09	
8.0.....	- .04	- .08	- .04	+ .09	+ .52	
10.0.....	+ .02	+ .06	+ .15	+ .28	+ .63	+1.02
12.0.....	+0.04	+0.13	+0.32	+0.54	+0.62	+0.60

COLOR-CURVES

The color-curve of an instrument is understood to indicate one which has for ordinates the focal settings for which the wave-lengths represented by the abscissae are in focus. Thus the color-curve of the 24-inch mirror, if it were drawn, would be a horizontal line, since a mirror possesses no chromatic aberration, reflecting light of all wave-lengths to the same focus. Spectra photographed with the 15° prism of 6-inch aperture attached to the 24-inch reflector are in good focus from λ 3500 to λ 6500.

40-INCH REFRACTOR

The color-curve of the 40-inch telescope as determined by Fox¹ is reproduced here (Fig. 9) for purposes of comparison. The ordinates increase with increasing distance from the lens. The long curve shows the behavior of the 40-inch lens alone, giving a flat region between λ 5000 and λ 6000.

¹ *Astrophysical Journal*, 27, 252, 1908.

The short curve, showing flat the region between λ 4000 and λ 5000, was obtained with the correcting lens (aperture 57 mm) in position. This lens is used for photographing stellar spectra and is placed in the cone of rays from the objective at a distance of 100 cm from the slit.¹

6-INCH UV DOUBLET

The material at hand for determining the color-curve of the 6-inch doublet was of two kinds: (1) focus plates taken with the grating R 6 referred to previously, and (2) focus plates with the 15° ultra-violet objective prism.

1. On two plates, taken with 4-inch aperture, for which the first part of Table XI gives the necessary data, the spectral distances (see page 9) to the right and left were measured in terms of revolutions of the micrometer-screw and converted to millimeters by the factor 1 rev. = 0.0364 mm. The mean values of these distances (ranging from 1.0 to 1.6 mm) furnished wave-lengths in best focus by means of the formula²

$$D = \frac{f\lambda}{a+d}$$

where

λ = wave-length of light used

f = focal length of lens

$a+d$ = grating space

D = distance from center of central image to spectral image of wave-length λ .

All these quantities are known except λ .

The first two plates gave very accordant results when the wave-length of best focus was plotted against focal setting. The mean curve is shown by the dotted curve of Figure 10.

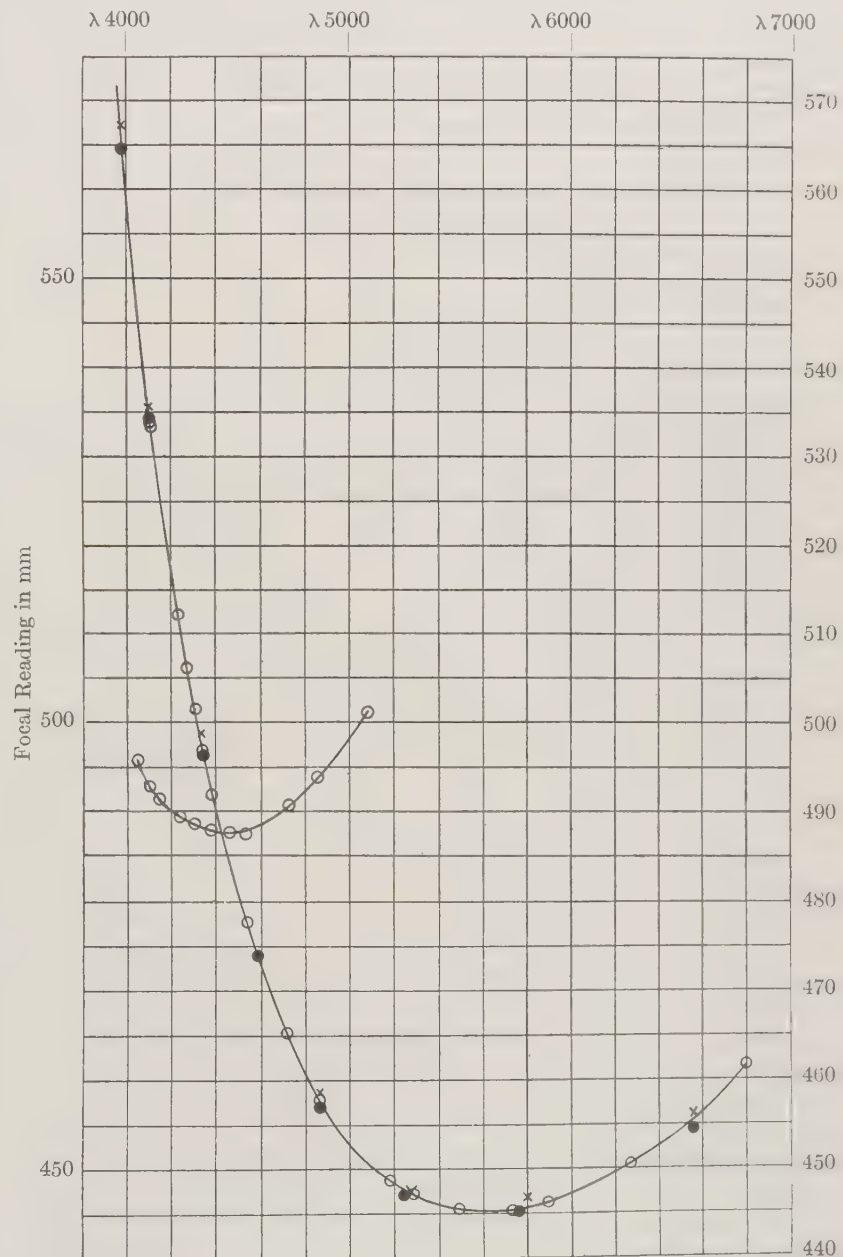


FIG. 9.—Color-curve. 40-inch refractor

¹ *Astrophysical Journal*, **15**, 12, 1902; also *Publications of the Yerkes Observatory*, **2**, 146, 1904.

² *Popular Astronomy*, **28**, 466, 1920.

2. The spectra obtained with the objective prism at different focal settings of the telescope carry their own scale of wave-lengths in the presence of the absorption lines. The prominent lines of the hydrogen series together with the H and K of calcium were the most useful. On twenty plates

TABLE XI
PLATES FOR UV COLOR-CURVE

PLATE No.	FIELD	GUIDED EXP. TIME	No. OF TRAILS	No. OF EXP. MEAS.	EXTREME FOCAL SETTING		KIND OF PLATE
					Inside	Outside	
UV 1192.....	Polaris	s 30		11	mm 30.0	mm 34.0	Seed 30
1193.....	α Cygni	20		13	30.0	34.0	Seed 30
OP 47.....	β Can. Min.	10	1	7	30.0	33.0	Seed 27
667.....	{ Spica	15	1, 2	5	30.0	33.0	Seed 23
	{ Polaris	15	1, 2	3	30.0	33.0	
668.....	{ Spica	15	2, 3	6	31.0	33.0	Seed 23
	{ Mars	15	1, 5	8	31.0	35.0	
669.....	{ Mars	15	1, 5	5	32.5	36.0	C.I.I.

covering a range of focus varying from 0.5 mm to 5.0 mm, estimates were made of the wave-length of the position of sharpest focus in the spectrum. These plates included spectra of Vega, Altair, β Canis Minoris, and Procyon on Seed 27 plates; of Betelgeuse and Sirius on Cramer Trichromatic and Isochromatic plates; and of Vega, Mars, and Antares on Wallace's Panchromatic plates.¹ The

range of wave-lengths included in these estimates covers λ 3750 to λ 6300. The temperatures at which the plates were taken varied from -4.7 to $+24.5$ C. The partial curves obtained from the various plates were shifted to agree with a mean focal setting of 33.0 mm for $H\beta$ at a mean temperature of $+12^\circ$ C. The mean curve derived from the twenty plates is shown by the dash line of Figure 10.

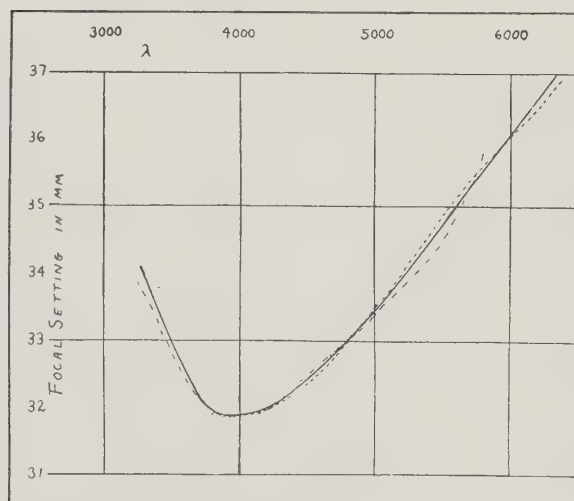


FIG. 10.—Color-curve, UV Doublet

position of best focus, when the trailed light had become too weak to affect the plate. The mean results from the four plates are given by the dot-dash line in Figure 10.

The three individual curves as shown in Figure 10 have been shifted vertically to bring the position of best focus to the mean focal setting 31.9 mm, corresponding to a mean temperature of $+17^\circ$ C. It was necessary to shift the curve obtained from the grating spectra toward the red by about 150 angstroms, to bring it into agreement with the other results. The final mean curve is shown by the full line. It indicates a flat region in the blue near λ 4000, in marked contrast to the flat region of the 40-inch lens in the green-yellow near λ 5500.

¹ *Astrophysical Journal*, 26, 317, 1907.

HARTMANN TESTS

By means of the Hartmann test,¹ light passing through different concentric zones of a lens is isolated by means of a suitable diaphragm and its effect is registered on a photographic plate placed successively inside and outside the focus of the instrument. The apertures in the diaphragm are placed diametrically opposite each other in a given zone and are made of such a size that their photographic images on a plate not at the focus resemble star-images in shape and neatness, and are measurable as such.

6-INCH UV DOUBLET

For the tests of the 6-inch UV lens, an aluminum diaphragm was used, containing forty apertures arranged at the ends of two perpendicular diameters in nine zones, and of two additional diameters in the seventh zone. Figure 11 is a reproduction of a pair of the patterns on plate UV 1893,

TABLE XII
HARTMANN TESTS

PLATE NO.	FIELD	EXP. TIME	NO. OF PAIRS OF EXP.	FOCAL SETTING		KIND OF PLATE (FILTER)	TEMP. C.	MEAN HOUR ANGLE
				Inside	Outside			
UV								
1753.....	Vega	m 10	3	mm 22.08	mm 42.02	Process	+29.8	h m +0 41
1893.....	Vega	10	2	22.00	42.10	Process	+ 1.5	-2 19
1889.....	Mars	7	2	21.98	42.18	CII, β 10	+ 7.0	+0 01
1892.....	Mars	8	2	22.00	42.09	CII, β 10	+ 2.0	+1 08
1899.....	Mars	10, 15	2	22.00	42.00	CII, β 10	+15.0	-0 12
1901.....	Arcturus	90, 75	1	21.95	42.05	CII, β 10	+13.9	+0 48
R								
3964.....	Vega	s 1/2, 1, 2	10	div. 73	div. 120 (390)	Process	+24.8	-0 40
3991.....	Vega	1/2, 1	8	73	120 (390)	Process	+17.8	-0 14

inside and outside the focus. The apertures were made with a drill 2.8 mm (0.113 inch) in diameter. The zones have radii of 7, 15, 23, 31, 39, 47, 55, 62, and 70 mm respectively. On plates taken 10 mm from the focus, the approximate diameter of the pattern is 2 mm. The plates used are shown in the first section of Table XII where the columns are self-explanatory.

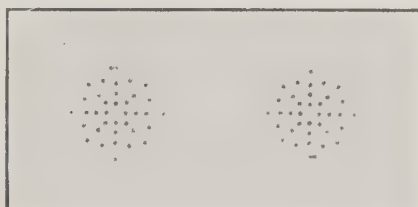


FIG. 11.—UV Hartmann patterns

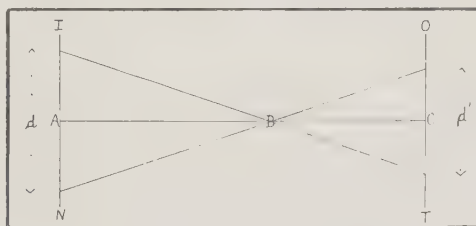


FIG. 12.—Theory of Hartmann test

The distances between diametrically opposite images on each zone were measured on the new screw measuring machine. The focal length of each zone is deduced from a comparison of the diameters of the zone when the plate is inside and outside the focus. In Figure 12, let d represent the diameter of a given zone on a plate in position IN , inside the focus, d' the diameter of the same zone on the plate at OT , outside the focus, B the point where the rays from the given zone cross, and AC the difference between the focal settings at IN and OT .

From similar triangles,

$$\frac{d}{d'} = \frac{AB}{CB} = \frac{AB}{AC - AB}, \quad \text{whence} \quad AB = \frac{d(AC)}{d + d'}.$$

The required focal setting for the zone is thus obtained by adding AB to the focal setting at IN .

¹ *Zeitschrift für Instrumentenkunde*, 24, 4, 1904.

The zonal foci obtained from the Process plates, which have excellent images, are shown in Table XIII. The position angles given in the third column are reckoned in a counter-clockwise direction from the starting-point near the extra aperture in Zone IX. Each focal setting recorded is that obtained from the mean of two perpendicular diameters. Since the images from Zones I and

TABLE XIII
ZONAL FOCI FOR UV DOUBLET ON PROCESS PLATES

Zone No.	Rad. of Zone	Position Angle	UV 1753				UV 1893			FINAL MEAN
			Exp. 1 and 6	Exp. 2 and 5	Exp. 3 and 4	Mean (+ .23)	Exp. 1 and 4	Exp. 2 and 3	Mean (- .24)	
	mm	°	mm	mm	mm	mm	mm	mm	mm	mm
I.....	7	45, 135	31.62	31.88:		31.71	32.24		32.24	31.96
II.....	15	0, 90		31.80:	31.77	31.78		32.26	32.26	32.01
III.....	23	45, 135	31.71	31.72	31.74	31.72	32.19	32.15	32.17	31.94
IV.....	31	0, 90	31.64	31.68	31.67	31.66	32.12	32.12	32.12	31.89
V.....	39	45, 135	31.60	31.63	31.64	31.62	32.04	32.08	32.06	31.84
VI.....	47	0, 90	31.68	31.65	31.66	31.66	32.14	32.12	32.13	31.89
VII.....	55	22.5, 112.5	31.71	31.71	31.72	31.71	32.18	32.18	32.18	31.94
VIII.....	62	67.5, 157.5	31.79	31.79	31.80	31.79	32.24	32.26	32.25	32.02
IX.....	70	0, 90	31.90	31.88	31.91	31.90	32.38	32.40	32.39	32.14

II overlap when both are exposed, they were each covered in turn, so that foci from these zones have less weight than the others. The values in the last column of Table XIII were obtained by shifting the curves from the two plates to a mean position, by application of the corrections indicated at the head of columns 7 and 10. The mean curve of the lower part of Figure 13 thus represents a

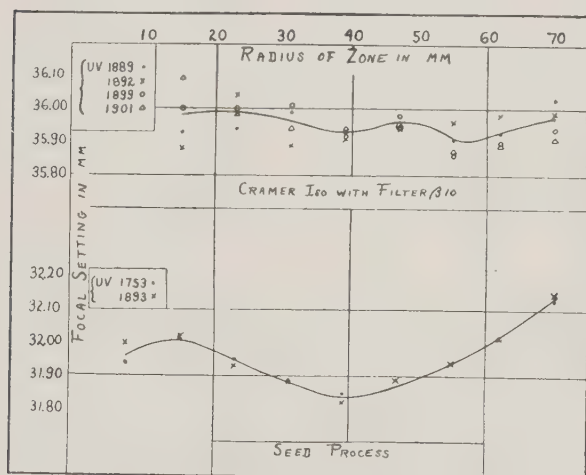


FIG. 13.—Curves of zonal foci for UV doublet

mean temperature of 15°6 C. The deviations of the focal settings from the separate pairs are very small, averaging 0.01 mm.

Three of the pairs of exposures were made with the diaphragm so placed that they were comparable for the indication of astigmatic errors, due to difference in curvature of the lens in different directions. The focal settings given in Table XIII are derived from the mean of diameters in two perpendicular planes. In Table XIV are tabulated the differences between these means and the individual values from the diameter in the position angle given in column 3, positive values indicating that the individual focal setting is greater than the mean. The errors derived from the diameters in the perpendicular planes are thus the same with

reversed sign. The mean errors are shown graphically in Figure 14, negative values inclosed in circles, all expressed in hundredths of a millimeter. The quantities are all small, but seem to indicate a slight increase of curvature along a diameter at position angle 135°.

The Seed Process plates are sensitive to a rather limited region in the blue lying between λ 3500 and λ 4500, with a maximum at λ 4000. An entirely separate region of the spectrum is utilized by the Cramer Isochromatic plates behind the yellow filter β 10. The filter absorbs the blue light and transmits the light of the yellow maximum (λ 5000–5800) of the Isochromatic plate. The four plates taken are shown in the second section of Table XII. The focal settings of the fifth and sixth columns really represent the position of the front of the filter. This consists of a gelatine film placed between

two pieces of glass. To obtain the position of the plate the reading must be increased by 3.2 mm, the thickness of the filter, and 0.30 mm, the thickness of two binding-strips. The images compare very unfavorably with those on the Process plates; they are less sharp and consequently overlap

TABLE XIV
ASTIGMATIC ERRORS FOR UV DOUBLET ON PROCESS PLATES

ZONE No.	RADIUS OF ZONE	POSITION ANGLE	UV 1753 3 AND 4	UV 1893		MEAN
				1 and 4	2 and 3	
I.....	mm 7	45°	mm	mm -0.105:	mm	mm -0.105:
II.....	15	0	+0.030		+0.055	+ .042
III.....	23	45	+ .015	+ .040	+ .030	+ .028
IV.....	31	0	- .020	- .025	- .020	- .022
V.....	39	45	- .005	+ .045	+ .045	+ .028
VI.....	47	0	+ .015	+ .045	+ .015	+ .025
VII.....	55	22.5	+ .025	+ .030	+ .030	+ .028
	55	67.5	+ .015	- .005	+ .000	+ .003
VIII.....	62	45	+ .005	- .005	+ .005	+ .002
IX.....	70	0	-0.010	+0.035	+0.015	+0.013

TABLE XV
ZONAL FOCI FOR UV DOUBLET ON CRAMER ISO PLATES WITH β 10

Zone No.	Radius of Zone	Position Angle	UV 1889 (- .03)	UV 1892 (- .09)	UV 1899 (+ .12)	UV 1901 (- .01)	Mean
	mm	°	mm	mm	mm	mm	mm
I.....	7	45, 135					
II.....	15	0, 90	35.96	35.97	35.88	36.10	35.98
III.....	23	45, 135	35.97	36.13	35.88	35.99	35.99
IV.....	31	0, 90	36.02	35.98	35.89	35.95	35.96
V.....	39	45, 135	35.96	36.00	35.80	35.95	35.93
VI.....	47	0, 90	35.98	36.03	35.86	35.96	35.96
VII.....	55	22.5, 112.5	35.94	36.05	35.74	35.88	35.90
	55	67.5, 157.5					
VIII.....	62	45, 135	35.96	36.07	35.78	35.90	35.93
IX.....	70	0, 90	36.06	36.08	35.82	35.92	35.97

TABLE XVI
ASTIGMATIC ERRORS FOR UV DOUBLET ON CRAMER ISO PLATES WITH β 10

Zone	Radius of Zone	Position Angle	UV 1889	UV 1892	UV 1899	UV 1901	Mean
	mm		mm	mm	mm	mm	mm
I.....	7	45°					
II.....	15	0	+0.115	-0.080	+0.055	-0.065	+0.006
III.....	23	45	- .060	- .010	+ .005	+ .160	+ .024
IV.....	31	0	- .105	- .105	- .070	- .110	- .098
V.....	39	45	- .015	- .005	- .020	+ .030	- .002
VI.....	47	0	+ .005	+ .040	+ .035	- .050	+ .008
VII.....	55	22.5	+ .015	- .015	.000	+ .020	+ .005
	55	67.5	.000	+ .005	+ .015	- .035	- .004
VIII.....	62	45	- .005	- .020	+ .030	- .005	.000
IX.....	70	0	+0.035	+0.005	0.000	+0.035	+0.019

more. It was impossible to measure Zone I on any of the plates. The results are tabulated in Table XV, which is of the same form as Table XIII, and plotted in the upper part of Figure 13. The latter represents a mean temperature of +9.5° C. The focal settings given represent the mean of two pairs of exposures on each plate, except UV 1901 which has only a single pair.

In spite of the poor quality of the plates and the lack of the close agreement between the separate plates characterizing the Process plates, the extreme range, leaving out Zone II which is subject

to the largest percentage of error because of its position, is 0.18 mm compared with 0.30 mm on the Process plates.

The astigmatic errors as shown by the Cramer plates are given in Table XVI and Figure 15. On the separate plates they average $+0.038$, as compared with $+0.026$ mm for the Process plates,

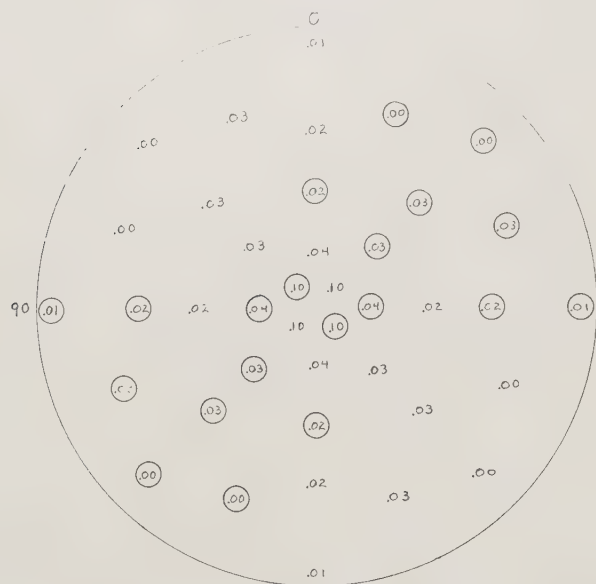
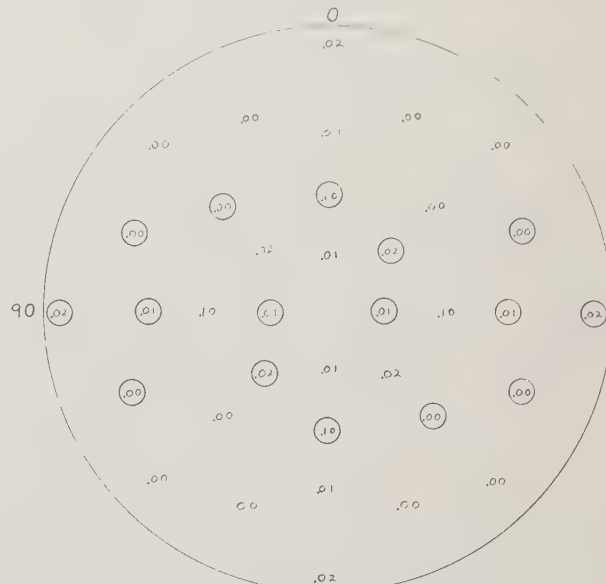


FIG. 14.—Process plates

FIG. 15.—Cramer Iso plates with filter β 10

but they are less systematic, as would be expected, so that the quantities shown in Figure 15 are smaller than those of Figure 14.

For determining the difference of focus for the two kinds of light, plates UV 1892 and 1893 are

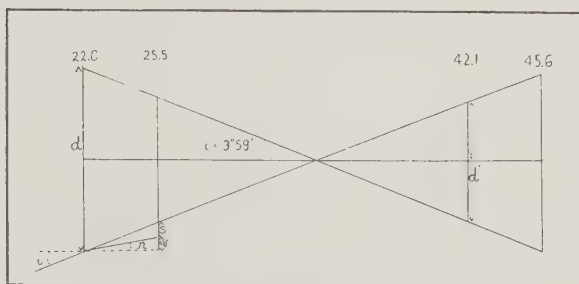


FIG. 16.—UV focus. Cramer Iso and Process plates

Cramer plate, at 25.5 mm and 45.6 mm, were first computed (Table XVII, column 4), whence, in Figure 16

$$s+q=0.2436 \text{ mm.}$$

The index of refraction of the filter was measured by means of a microscope,¹ whose vertical motion was measured by means of an inside caliper. For Zone VII the angle of incidence i is $3^\circ 59'$, whence r , the angle of refraction, is $2^\circ 34'$. The thickness of the filter being 3.2 mm,

$$q=3.2 \text{ mm } (\tan 2^\circ 34')=0.1433 \text{ mm}$$

whence $s=0.1003$ mm, the increment in the radius of Zone VII due to refraction through the filter.

¹ Watson, *Textbook of Physics*, p. 494.

The radius of Zone VII outside the focus is correspondingly decreased, so that we may apply the corrections for refraction (2s) to the diameters of column 4 and obtain those of column 5; and thence calculate the focal setting. The last line of Table XVII gives the focal setting computed from the diameters just above, and indicates that 1.44 mm of the total increase in focal setting between UV 1893 and UV 1892 (3.87 mm) may be laid to refraction, leaving 2.43 mm to be accounted for by the color-curve (Fig. 10). Assuming λ 4000 as the maximum on the Process plate, an increase of 2.43 mm in focal setting points to λ 5400 approximately as the maximum for the combination of Isochromatic plate with filter. This is near the maximum as shown by tests with the Wallace spectrograph.

TABLE XVII

UV 1893 PROCESS		CALCULATED DIAMETER			UV 1892 C.I.I., β 10	
Focal Setting	Diam. of Zone VII	Focal Setting	Refraction		Focal Setting	Diam. of Zone
			Uncorrected	Corrected		
mm	mm	mm	mm	mm	mm	mm
22.0.....	1.4186	25.5.....	0.9304	1.1310	22.0.....	1.4636
42.1.....	1.3828	45.6.....	1.8690	1.6684	42.1.....	1.3235
Focus for Zone VII.....	32.18		32.18	33.62		36.05
Δ Focus from 32.18.....			0.00	1.44		3.87

24-INCH REFLECTOR

The Hartmann tests of the 24-inch mirror were made by means of two diaphragms R 4 and R 5, made from bristol board, each used in two positions 45° apart. In this way nine zones were tested, four diameters of each zone except the first being measured. As the result of various experiments, it seemed best to vary the diameter of the circular apertures from 2.5 cm for the inner zones to 3.5 cm for the outer zones. The plates used are shown in the last section of Table XII. They were taken about 7 mm from the focus, showing patterns about 1.6 mm in diameter. The images are in general not as excellent as those given by the 6-inch lens on Process plates.

The focal settings obtained are shown in Table XVIII, where they are expressed in terms of

TABLE XVIII
ZONAL FOCI FOR 24-INCH REFLECTOR ON PROCESS PLATES

ZONE No.	RAD. OF ZONE	R 3964						R 3991			FINAL MEAN
		Exp. 1 and 16 6 and 11	Exp. 2 and 17 7 and 12	Exp. 3 and 18 8 and 13	Exp. 4 and 19 9 and 14	Exp. 5 and 20 10 and 15	Mean	Exp. 3 and 16 5 and 9	Exp. 4 and 17 6 and 10	Mean (+7.9)	
I.....	76	div.	div.	div.	div.	div.	div.	div.	div.	div.	div.
II.....	102	236.1	238.2	237.9	235.5	235.7	237.4	229.5	230.0	229.8	237.4
III.....	127	236.0	237.3	236.4	235.5	235.7	236.2	229.5	230.0	229.8	236.7
IV.....	152	234.9	234.1	234.5	234.3	234.0	234.4	226.1	226.3	226.2	234.3
V.....	178	235.2	235.7	234.3	234.6	233.9	234.7	224.8	226.0	225.4	234.2
VI.....	203	237.3	237.8	237.3	237.9	237.2	237.5	230.2	229.7	230.0	237.6
VII.....	229	238.1	238.0	237.5	238.4	237.6	237.9	229.7	229.3	229.5	237.7
VIII.....	254	240.5	239.9	238.8	239.8	240.1	239.8	232.9	232.2	232.6	240.0
IX.....	279	241.9	241.6	242.0	241.3	241.1	241.6	232.5	233.4	233.0	241.4
		241.9	241.9	240.1	240.8	241.3	241.2	233.9	234.8	234.4	241.6

divisions of the focusing circle. (One division corresponds to 0.044 mm.) Each column gives the means derived from two pairs of exposures. The quantity at the head of the eleventh column was added to the quantities below it before they were combined with the means of column eight to give the "Final Mean."

The means from the two plates are plotted in Figure 17, where for purposes of comparison the ordinates are expressed both in terms of divisions of the focusing circle and mm of focal length (assuming 2360 mm equivalent to the focal setting 238). A rotation of one division of the focusing circle changes the focal setting 0.044 mm. The curve is somewhat similar in form to the lower curve of Figure 13, with practically the same range of 0.30 mm.

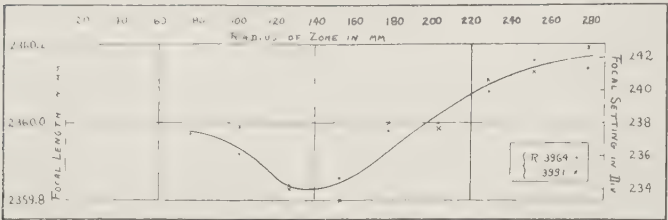


FIG. 17.—Curve of zonal foci for reflector. Process plates

have an average value of 0.028 mm. The quantities in the figure represent thousandths of a millimeter.

The reflector being achromatic, it was not thought necessary to duplicate the Hartmann tests on Isochromatic plates with the Jena glass filter J 1, used for photovisual work. It is interesting to note, however, that the increase in focal setting (Δf) for plates exposed through the filter, found necessary and determined upon by means of focus plates, is almost exactly accounted for by the increase due to refraction through the filter as computed from the index of refraction (μ) of the glass. From measures with a microscope, $\mu = 1.47$, whence $\Delta f = 0.87$ mm. As determined from focus plates, $\Delta f = 20 \text{ } d = 20 \times 0.044 \text{ mm} = 0.88 \text{ mm}$.

TABLE XIX
ASTIGMATIC ERRORS FOR 24-INCH REFLECTOR ON PROCESS PLATES

Zone No.	Radius of Zone	Position Angle	R 3964	
			div.	mm
I.....	76			
II.....	102	0°	+0.43	+0.019
		45	+ .65	+ .029
III.....	127	0	+ .21	+ .009
		45	+ .67	+ .030
IV.....	152	0	+ .05	+ .002
		45	— .57	— .025
V.....	178	0	— .52	— .023
		45	+ .59	+ .026
VI.....	203	0	+ .89	+ .039
		45	+ .44	+ .019
VII.....	229	0	— .75	— .033
		45	+ .58	+ .026
VIII.....	254	0	— 1.62	— .071
		45	— 0.52	— .023
IX.....	279	22.5	— 0.40	— .018
		67.5	+ 1.31	+ 0.058

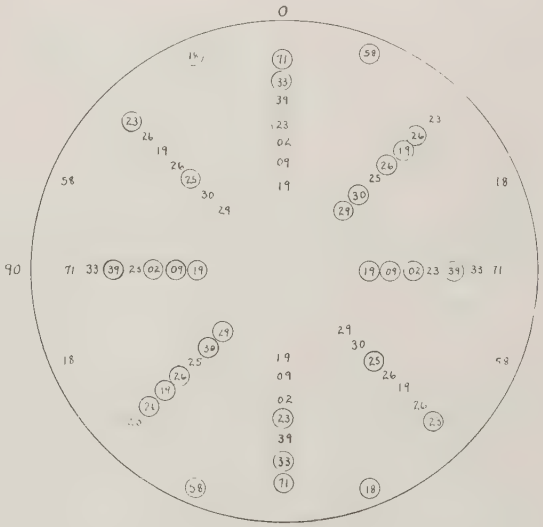


FIG. 18.—Astigmatic errors of reflector. Process plates.

40-INCH REFRACTOR

The lens of the 40-inch telescope was tested by Fox in 1908. The results are given in the *Astro-physical Journal*, 27, 237, 1908.

Figure 19 shows, for comparison, the results of the Hartmann tests of all three telescopes.

THE HARTMANN T

In order to determine the Hartmann T ,¹ it is first necessary to find the plane F_0 where the cone of light from all zones has the least diameter. This will ordinarily be the mean of the greatest and least focal lengths obtained from zones nearest the edge of the lens weighted according to the radius

¹ *Zeitschrift für Instrumentenkunde*, 24, 46, 1904.

of the zone (r). The diameter (d) of the cone of rays from each zone in this plane F_0 is then given by $d = \frac{2r(F-F_0)}{F_0}$ where F is the focal length for the zone of radius r . The Hartmann T is a mean value of d , weighted according to the light-gathering power of the different zones, and expressed in hundred thousandths of the focal length

$$T = \frac{100,000}{F_0} \frac{\sum rd}{\sum r} = \frac{200,000}{F_0^2} \cdot \frac{\sum r^2(F-F_0)}{\sum r}$$

Table XX gives the values of T obtained for the three telescopes with various apertures, a mean value for the 40-inch being taken from Fox's paper referred to above.

TABLE XX
THE HARTMANN T

6-INCH UV DOUBLET		24-INCH REFLECTOR		40-INCH REFRACTOR	
Aperture	T	Aperture	T	Aperture	T
in.		in.		in.	
6	1.41	24	0.73	40	0.20
4 (Process)	0.28	18	0.52		
4 (CII, β 10)	0.13	12	0.21		

Hartmann's classification puts the 40-inch, 4-inch aperture on the UV doublet, and 12-inch aperture on the reflector into the class "Very good." These are the apertures actually being used at the present time in photometric work. The smallest value of T , derived from Isochromatic plates with filter, used with 4 inches on the UV doublet, is satisfactorily confirmed by the excellence of the stellar images on these plates.

SHAPE OF THE CALIBRATION CURVES

We are now in a position to discuss the form for the preliminary curves of Figures 2, 5, and 6. In general, their shape will depend on (1) how much and in what manner the photographic plate, by reason of its being a plane, departs from the focal surface of the lens or mirror, which is spherical,¹ (2) the aberrations of the lens or mirror, (3) the distribution of light in the cone, and (4) the limiting sensitiveness of the plate.

24-INCH REFLECTOR

The curves for the reflector (Fig. 2) show for the brighter stars the image increasing in size with distance from the optical axis. This would be expected from the fact that the image formed by a parabolic mirror is theoretically a point only for an object on the axis.² Elsewhere in the field the aberration is proportional (1) to the angular distance of the object from the axis, and (2) to the ratio of the aperture to the focal length of the instrument. The first explains the form of the individual curves, and the second the fact that the curves are less steep for the 12-inch aperture than

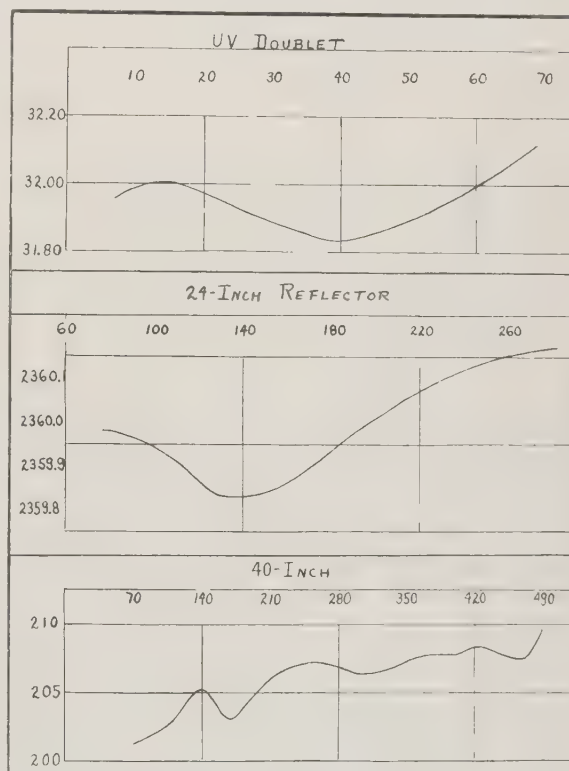


FIG. 19.—Curves of zonal foci. Abscissae: Radius of zone in mm. Ordinates: Focal setting in mm.

¹ C. L. Poor, *Astrophysical Journal*, 7, 120, 1898.

² *Ibid.*, p. 119.

for the 18-inch. The aberration does not produce an image symmetrical about its center (see Plate VI, showing the growth of images in the reflector field), but irradiation makes the photographic image practically round at a considerable distance from the axis. The curves grow less steep with decrease in size of the image until at $S(R\ 5) = 8.5$ for the yellow light on the Cramer plates, and at a point about two scale-units fainter for the blue light on the Seed plates, the images are practically the same size for some distance out from the axis; this must be attributed to the non-uniform distribution of intensity in the cone of light forming the image, combined with the law of growth in the sensitive film.

When the preliminary curve is a horizontal line, light from the outer parts of the cone is below the intensity necessary to produce blackening of the plate and the image does not increase in size. For still fainter images, some of the light which is intense enough to affect the plate and form part of the image of a star on the axis, falls below the requisite intensity for a star off the axis, and the size of the image actually decreases with distance from the axis. This falling-off in size occurs for a brighter image on the Cramer plates than on the Seed presumably because the latter utilize a longer range of wave-length than the former, so that a given limiting intensity in the cone of light will be reached sooner in the yellow light than in the blue.

The extreme range in focal length for the different zones on the reflector is shown in Figure 17 to be 0.32 mm. Assuming that the focus adopted for the plate were the minimum value shown by the curve, the radius of the cone formed by the converging rays not yet brought to a focus is, for an aperture of 18 inches and a star not more than 30' from the axis, 0.031 mm (2".7). The value of the aberration (A) under these same conditions, as computed from the expression¹

$$A = \frac{1}{16} R^2 a'' \text{ where } R = \text{the ratio of aperture to focal length, and} \\ a'' = \text{the angular distance of the object from the} \\ \text{axis in seconds of arc}$$

without projection from the true focal surface to the plane of the plate, is of the order of 13". Therefore, it is evident that the variation in focus for the different zones is of little importance, in the case of a plate carefully focused, compared with the aberration demanded by theory. Actually, using the upper preliminary curve for the Seed plate taken with 18-inch aperture in Figure 2, the image changes from a diameter of 9" on the axis to 13" at an angular distance of 30' from the axis.

40-INCH REFRACTOR

The preliminary curves for the 40-inch indicate a very slight, gradual increase in the size of the image with increase in distance from the axis. This is the effect that would naturally be expected as the focal surface of the lens departs from the plane surface of the photographic plate. The faint images show a decrease in size as r increases, due evidently to the fact that the intensity of the light in the outer part of the cone falls below the limit of sensitiveness of the plate.

6-INCH UV DOUBLET

The preliminary curves of Figure 6 indicate that images of various sizes do not come to their best focus at the same distance from the optical axis. The curves as adopted show that an image of average size [$S = 6$ in terms of UV 5(442)] is in best focus about 2° from the axis; a small one ($S = 10$), on the axis. In consequence we must assume different focal surfaces, intersecting the plane of the photographic plate at different places, for images of different sizes. Figure 7 indicates that a decrease in focal setting causes the point of best focus to move away from the axis and the central ascent of the curve to increase in steepness. Therefore, we may account for the form of the preliminary curve for a bright star by assuming that the plate is situated as shown in Figure 20, where

¹ *Ibid.*, p. 120.

PP gives the position of the plate, BB the focal surface for a bright star, FF the focal surface for a faint star, DD the focal circle for the bright star, and T the position of good focus for the faint star.

The images for which BB is the focal surface are smallest in the focal circle DD , and grow larger as their distance from it increases in either direction. The surface FF is tangent to the plane of the plate PP , so that the corresponding faint images are in best focus at the center and increase in size as their distance increases. The faintest images decrease again in size when they reach a distance of 2° or 3° from the axis, presumably because rays from the outer part of the cone of light become too weak to affect the plate. Stars intermediate in brightness find their focus in surfaces between FF and BB . Measures of the central images on plates UV 1192 and 1193 (Table XI), confirm the information given by the calibration curves as to the position of the plate and give an idea of its distance (ST) from a position of tangency to the focal surface BB . The diameters of the central

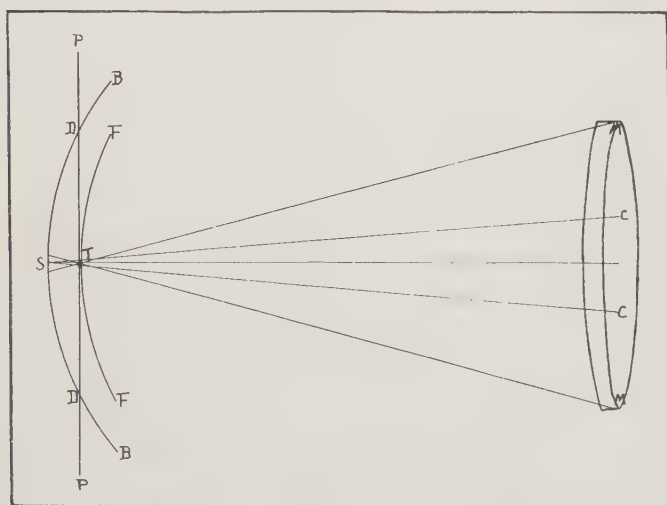


FIG. 20.—Focal surfaces for UV doublet

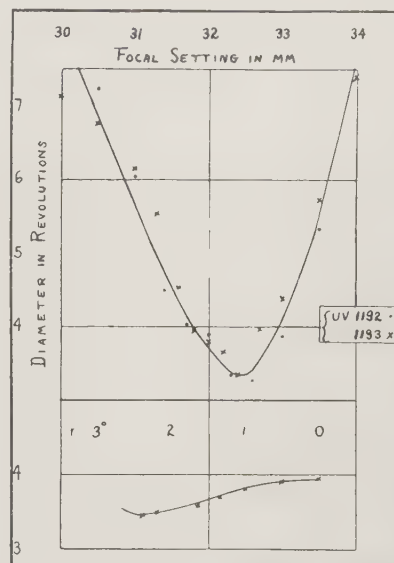


FIG. 21.—Central images on UV focus plates. Grating R 6.

images are plotted in terms of revolutions of the micrometer screw as ordinates against focal settings in mm as abscissae (Fig. 21). The plates were taken at practically the same temperature ($+15.7^\circ\text{C.}$) and the images at the center were almost alike in size so that the mean curve represents both plates very well. The diameter of the image at focal setting 31.84, corresponding to the temperature $+15.7^\circ\text{C.}$ on the focusing schedule used with the UV telescope, is $3.95r$ (r = a revolution of the micrometer screw). It changes to $3.35r$, its smallest value, when the focus is 32.40, and back again to $3.95r$ at focus 32.87. The distance ST is thus 0.56 mm. The typical calibration curve, plotted to the same scale, for an image of diameter $3.95r$ at the center, shows the image with a diameter of about $3.40r$ at a distance from the axis of 2.4° (i.e., TD in Figure 20 = 34.1 mm, since $1^\circ = 14.21$ mm). Starting from S as an origin, we thus have the co-ordinates (0.56 mm and 34.1 mm) of a point on the focal surface where it intersects the plate. Other points may be deduced from the curves of UV 1484 (Fig. 7) which represent an image of practically the same size as that we have been considering. Table XXI gives the data from a smooth curve connecting values of focal setting and r of best focus, and the derived values of ST and TD (Fig. 20) in mm. Neither a spherical nor parabolic surface fits the points given. The points along SD are best represented by a straight line through S , inclined to the perpendicular to the optical axis $57'$ of arc. The radius of a spherical

surface containing the points S and D , where D is represented by the co-ordinates 0.56 mm and 34.1 mm would be 1038 mm (compare the focal length of the objective: 814 mm).

The explanation of the difference in focus for images of different size lies in the combination of the aberrations of the lens with the limited sensitiveness of the photographic plate and the distribution of intensity of the light in the cone from the lens. Call the smallest intensity which produces blackening of the silver grains of the plate its threshold intensity. The Hartmann test (Fig. 13) shows that the lens with 4-inch aperture has positive aberration, i.e., the marginal rays come to a focus nearer the lens than the central rays. This is confirmed by the appearance of extra-focal images on plates taken 6 mm inside the focus. These images are dense around the edge with a less illuminated region in the center. The focus plates UV 1192 and 1193 (Table XI) show the same appearance for central images inside the focus, and the reverse for those outside. These phenomena indicate positive aberration.¹

Exposures made through the diaphragm for the Hartmann test, too short to give images measurable with the artificial scale, give images which may be compared with regard to density. These

TABLE XXI

Focal Setting	ΔS from 32.36	ST	r of Best Focus = TD	
mm	mm	mm		mm
32.1.....	-0.26	0.82	3.4	48.31
32.2.....	- .16	0.72	3.0	42.63
32.3.....	- .06	0.62	2.6	36.95
32.36.....	.00	0.56	2.4	34.10
32.4.....	+ .04	0.52	2.25	31.96
32.5.....	+ .14	0.42	1.85	26.29

show that the distribution of intensity in the cone of light is practically uniform. Differences in intensity of the light which strikes the plate may thus be ascribed only to differences in distance from the focus, the separate cones overlapping and reinforcing each other more and more as the focus is approached. The Hartmann curve of zonal foci indicates that our settings on the focusing schedule correspond to the focus for the zones of shortest focal length, i.e., the focus for the marginal rays. The behavior of the bright and faint stars is, then, accounted for if (in Fig. 20) we assume that FF is the focal surface for the marginal rays, and that the central rays out of focus in this plane are all below the threshold intensity of the plate so that the marginal rays alone are effective in forming the faint images. As brighter and brighter sources of light are employed, and the images increase in size, the central rays gradually become effective in focal surfaces which may be imagined to lie between FF and BB . We have seen already that the distance ST is in the neighborhood of 0.5 mm. The Hartmann curve is very uncertain inside a radius of 15 mm, so that it is difficult to say just which zones come to a focus in the surface BB , but in general we may assume that the bright stars send light enough to bring the intensity from any zone here above the threshold value for the plate. It is noticeable that the preliminary calibration curves on these photographic plates do not become horizontal, as in the cases previously discussed.

When allowance is made for the difference in the gradation between scales UV 5(442) and 6(442), the preliminary curves on the photovisual plates are quite similar to those on the photographic. As on the reflector, the photovisual curves become horizontal while the photographic are still curving strongly. Owing to the character of the images in the Hartmann test on the photovisual plates, the focus of the two inner zones is unknown. It may be long enough to account for the form of the preliminary curves in a manner entirely analogous to that already discussed.

¹ Bigourdan, *Les Méthodes d'Examen des Lunettes et des Télescopes*, p. 113; Bell, *The Telescope*, p. 208.

THE DETERMINATION OF MAGNITUDES

It is convenient to divide the process of determining magnitudes from a photographic plate into three principal steps:

1. The measurement of the images with an appropriate scale, followed by such correction of the measures as is necessary to make the series as uniform as possible and free from all known systematic errors;
2. The application of a magnitude-scale or measuring-rod of known length, calibrated in terms of stellar magnitudes, to the system of scale-readings established in 1, in such a manner as to convert it into a system of relative magnitudes;
3. The conversion of the system of relative magnitudes, by the use of the known magnitude of one member or preferably of a group of members of the series, into a system which bears a definite relation to other published systems.

The apparatus and method of measurement of images has been described. The outstanding correction for systematic errors necessary under 1 is due to distance from the optical axis. Tables containing corrections of this nature have already been derived and discussed.

The technique of planning exposures in such a manner as to establish on all plates a means of locating the zero-point referred to in 3, whether by duplicate exposures on standard fields, or by means of intermediate standards, is a problem to be settled separately for each special program of investigation. This question will not be considered further in this paper.

MAGNITUDE-SCALE FROM THE OBJECTIVE GRATING

The determination of a standard magnitude-scale (referred to in 2), however, is a problem of general application to all photometric work. The standard sought must be one which does not change its behavior for stars of different color, photographic plates of different quality, and if possible for stars of different intensity. The proportion of light from a given source which suffers diffraction into spectral images on passing through a parallel-wire grating is theoretically a constant as regards the three conditions mentioned above and is therefore a convenient standard to use. For the theory regarding the formation of spectra by diffraction, as a consequence of the wave-theory of light; the derivation of the formulas connecting the intensities of these spectral images with that of the central image formed simultaneously, and of the free image formed by an equal exposure without the grating; and the practical design of the grating so that the spectra shall be round and measurable like stellar images, and of appropriate intensity—the reader is referred to my recent paper in *Popular Astronomy*, **28**, 326, 458, 1920. Attention is especially called to Figure 22 (reproduced from that article) which shows at a glance the theoretical grating constants, i.e., the difference in magnitude between free and central, or central and spectral images, to be obtained from a given ratio between the aperture (a) and sum of aperture and opaque region ($a+d$ = “grating space”).

The grating constants derived from the formulas are in all cases checked by exposures on fields (such as the cluster in Perseus, the Pleiades, the North Polar Sequence) containing stars of known magnitude. It was suggested among the advantages of the “normal” grating (where free space equals opaque space) that Curve 3 (Fig. 22) changes slope in the region of $a/a+d=0.5$ so slowly that the difference in magnitude between free and spectral images ($=2^m48$) may be regarded as well established, when the measures of a and d are of only moderate accuracy. This property of the normal grating is of use in checking the scale of magnitudes available in the particular field used (such as the Münch photographic scale for the Pleiades). If sufficient exposures—(1) free, (2) with normal grating, (3) with unknown grating—are made to render safe the comparison of free and spectral images of (1) and (2), the normal grating thus furnishes data for the accurate determination of the unknown constants of the other grating.

Table XXII gives the data concerning the gratings in recent use on the three telescopes, illustrative of the principles discussed above. The columns are self-explanatory up to the sixth and seventh, which give the width of the free space and of the opaque space. The eighth gives the ratio which is the abscissa for the curves of Figure 22. The ninth and tenth give the distance between the centers of central and spectral images for the photographic rays (calculated for $\lambda=4100$) and for

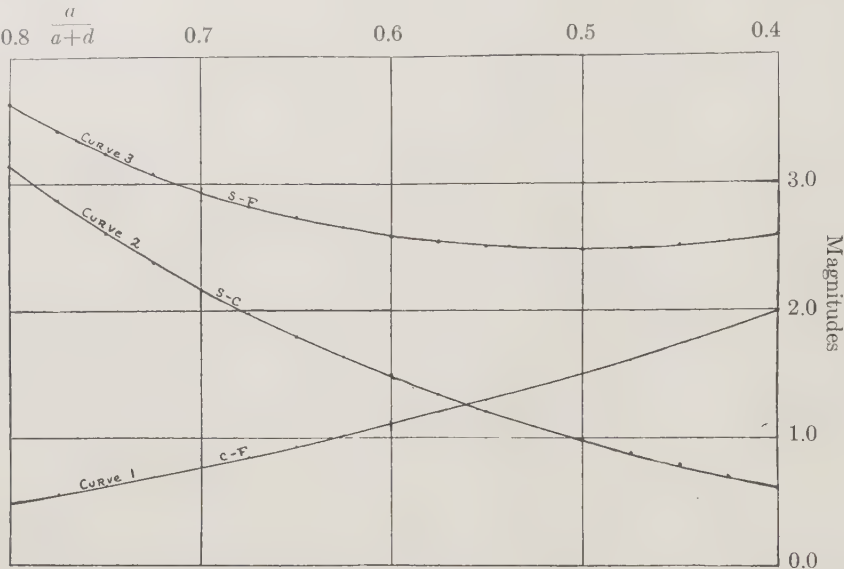


FIG. 22.—Effect of objective grating

TABLE XXII
PHOTOMETRIC GRATINGS

Telescope	Name	Aper- ture	Material	No. of Rods	<i>a</i>	<i>d</i>	$\frac{a}{a+d}$	<i>D(ph)</i>	<i>D(pv)</i>	Abs.	Magnitude Interval	
					mm	mm		mm	mm	$\frac{c-f}{M}$	$\frac{s-c}{M}$	$\frac{s_s-c}{M}$
UV.....	R 6	4	B & S Wire Gauge No. 36	401	0.124	0.129	0.490	1.32	1.80	1.57	(0.92)	
	R 8	4	B & S Wire Gauge No. 18	52	1.039	1.013	0.507	0.16	0.22	1.50	0.98	
	R 9	4	B & S Wire Gauge No. 19	32	2.281	0.923	0.712	0.10	0.14	0.74	2.26	
	P 10	4	Stubbs' Drill Gauge No. 50	28	1.715	1.766	0.493	0.10	0.13	1.53	0.95	
	S 1	4	Stubbs' Drill Gauge No. 60	27	2.819	0.991	0.74	0.09	0.12	(0.64)	2.53	
R.....	P 7	17.5	Stubbs' Drill Gauge No. 23	56	3.87	3.93	0.496	0.12	0.17	1.50	0.99	
	P 7b	17.5	Stubbs' Drill Gauge No. 23	21	3.87	3.93	0.496	0.12	0.17	(0.35)	3.33	
	P 7b	12	Stubbs' Drill Gauge No. 23	21	3.87	3.93	0.496	0.12	0.17	(0.53)	2.80	
40-inch.....	P 5	40	Wooden Slats	40	12.3	12.7	0.491	0.32	0.45	(1.53)	0.95	3.14
	P 6	40	Wooden Slats	23	12.3	12.7	0.491	0.32	0.45	(0.78)	2.38	5.03

photovisual rays (calculated for $\lambda=5600$). The last three columns give the mean values of the grating constants as adopted for use; the quantities in parentheses depend on calculation only and are not in common use. The magnitude-interval between central image and third-order spectrum tabulated in the last column is useful for bright stars on the 40-inch plates.

Table XXII is interesting because of what may be read between the lines regarding difficulties and correction thereof with consequent progress. The large dispersion of R 6, or difference between $D(Ph)$ and $D(Pv)$, points to the usefulness of its elongated spectra for determining the effective focal setting of plates taken behind it, already discussed. The spectral images formed by R 8

were elongated, because of too great dispersion; and P 10 is the new normal grating constructed of coarser wire, with $D(P_h)$ and $D(P_v)$ consequently smaller. This gives round images. The spectra of R 9 were elongated, presumably because they were so faint (note the large value of the magnitude-interval) that the photographic action as a rule did not round them out. S 1 represents a construction which should give constants similar to those of R 9, but make the dispersion less and so reduce the tendency of the spectra to be linear. This was successful in a measure, though the range of measurable pairs of central and spectral images is seriously limited where the two lie so close together.

A serious practical difficulty in the case of both the reflector and the 40-inch lies in the finding of guiding-stars in the right position, bright enough for use with the normal grating, which has an absorption of a magnitude and a half. Under ordinary conditions, guiding on the 40-inch demands an 11^m0 star, 18 inches on the reflector a 9^m0 star, and 12 inches on the reflector a 7^m5 star. In order

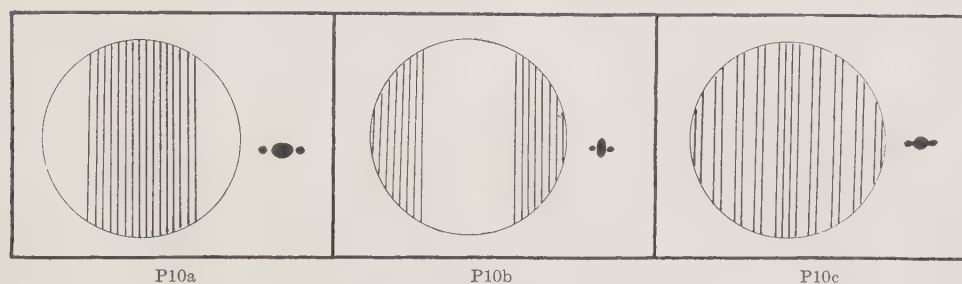


FIG. 23.—Partial gratings for UV doublet

to preserve the advantages of the normal grating (namely, absence of spectra of even order and maximum brightness of those of odd order) and at the same time admit more light into the central image for purposes of guiding, the plan of using a partial normal grating was adopted. The grating P 7 on the reflector was changed into P 7b, which has been used extensively with the 12-inch aperture, and the 40-inch P 5 was transformed into P 6 by removal of an equal number of rods from each side of the aperture, leaving slightly more than half the original number across the center. The constants of both gratings were recomputed on the basis of the facts that (1) half the light which strikes the covered portion of the aperture never gets through, (2) half the light passing between the bars goes into the central image, since the absorption of 1^m50 shows that the central image receives a quarter of the incident light, i.e., a half of the half of the incident light that gets between the bars, (3) all the light that strikes the uncovered parts of the aperture goes into the central image. Measurement of the relative areas for grating P 7b must take into account the area cut out by the flat mirror.

The attempt to use a similar partial grating on the 6-inch telescope was not successful. The sketch (Fig. 23) shows the different forms tried with the resulting shape of image. In each case, the light which passes unhindered through the outer parts of the lens into the central image causes its elongation because light through portions 90° away suffers diminution.

In the case of the images obtained through the partial grating on the 40-inch, an analogous, though not quite similar, phenomenon is noted. The central images of stars off the axis do not lie midway between the spectral images, the displacement being toward the center (not radially, but parallel to the length of the diffraction pattern) and increasing with distance from a central line. This displacement does not occur when a complete grating is used. The investigation of this phenomenon is not yet complete. Briefly, however, measures of free, central, and spectral images on plates taken with partial gratings and plates taken with complete gratings indicate that the positions of spectral images obtained from a partial grating show a systematic shift, in the direction of

the dispersion of the grating, compared with those taken with a complete grating. Measures of plates containing exposures free, and with both partial and complete gratings, remain yet to be made.

The results in general from the partial gratings show that their constants should be carefully checked by comparison with normal complete gratings, as has been done for the three instruments.

The constant of the grating once adopted, the conversion of the system of scale-readings into a system of magnitudes by means of this constant takes place according to the well-known principles for calibrating any scale. In this instance, instead of having two fixed points and an unknown measuring-rod, as is often the case, the grating constant acts as a measuring-rod of known length and only one fixed point, the zero-point, is necessary. The variation of the value, in terms of magnitudes, of one unit of the scale used, as related to the condition of the seeing when the plate was taken

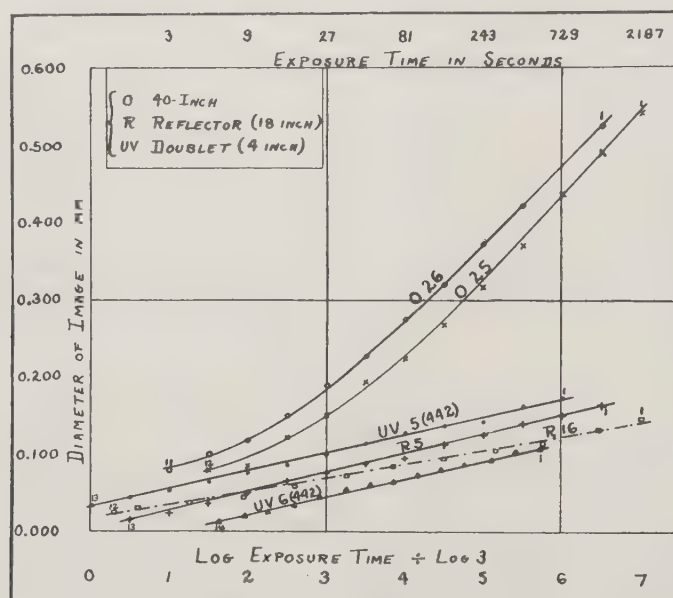


FIG. 24.—Artificial scales

and the resulting quality of the plate, is a matter of interest. It is a fact that the value of a scale-unit is systematically smaller than the average, when plates are taken in poor conditions of seeing, and systematically larger than the mean value, when conditions are unusually fine. This seems to be due to the fact that unsteadiness of the atmosphere diminishes the density of the fainter images to a greater degree than it tends to make their diameters larger, so that the estimate makes the star fainter. The correlation between conditions of seeing and constancy of the scale-value throughout the range of magnitude on the plate is difficult to determine, because in so many cases the use of a constant scale-value indicates lack of sufficient data to establish a progressive variation. It is true that out of a group of 39 photographic plates on the reflector (these plates giving most abundant data for determining the scale-value), only 50 per cent permitted the use of a constant multiplier, as compared with 85 per cent (out of 23 plates) and 81 per cent (out of 51 plates) for the reflector photovisual and the 40-inch plates respectively. The last two columns of Table XXIII give for five of the scales used a mean value of the scale-unit with the number of plates from which the mean was taken.

A STUDY OF THE ARTIFICIAL SCALES

In order to indicate the actual sizes of the images produced by the three telescopes and used in the measurement of the plates, measures of the diameters of the two scale-rows most used with each instrument are shown in Figure 24, plotted with the logarithmic scale of exposure-times as abscis-

sae. Table XXIII gives for the scales measured the name, number of plate from which it was cut, kind of plate, name of color-filter, star at which the exposure was centered with its magnitude, photographic or photovisual as the case may be, and the factor by which each exposure-time was multiplied to obtain the next succeeding. The last two columns of the table have already been explained.

Comparing the three photovisual scales O 26, R 5, and UV 6(442), the values of the increase in diameter of the image corresponding to a doubling of the logarithm of the exposure time are 0.220 mm, 0.064 mm, 0.060 mm, respectively. It might be expected that the precision of the magnitudes

TABLE XXIII
ARTIFICIAL SCALES

Scale	Plate No.	Kind of Plate, Filter	Star	Mag.	Exp. Ratio	Mean Scale Value	No. of Plates
UV 5(442)	UV 1328	Seed 30	B.D. +85°19	5 ^m 5	√3	0 ^m 58	31
UV 6(442)	UV 1310	C.I.M., β 10	B.D. +85°19	4.2	√2	0.28	19
R 16	R 3805	Seed 30	B.D. +88°104	8.2	√4	0.61	59
R 5	R 3627	C.I.M., J 1	N.P.S. No. 5	6.4	√3	0.44	31
O 25	O 313	{Stanley Com., "Dec. '07"	λ Urs. Min.	6.4	√3	0.53	46
O 26	O 315	C.I.M., "Dec. '07"	λ Urs. Min.	6.4	√3		

obtained would show a correlation with these values. As a matter of fact, the magnitudes on separate plates of the stars in six of the Kapteyn fields show an average residual from the mean of $\pm 0^m.09$ for both the reflector and the 40-inch plates. This value was derived from intercomparison of, on the average, 2.4 reflector plates and 3.1 40-inch plates, so that the R plates show a slight advantage. The total number of stars included in each series was 170.

Table XXIV gives for the six scales the approximate diameter of the smallest image used, the

TABLE XXIV

Scale	Diam. of Smallest Image	Diffraction Disk		Mea. Diam of Cone
O 26	mm 0.079	0'.112	mm 0.010	mm 0.033
O 25	0.079			
R 5	0.015	0.250	0.003	0.012
R 16	0.025			
UV 5(442)	0.015	1.125	0.004	0.002
UV 6(442)	0.015			

theoretical diffraction disk in seconds of arc and in mm, and the mean diameter in mm of the cones of light from various zones of the mirror or lens weighted according to the radius of the zones. The figures given are for 18 inches on the reflector and 4 inches on the doublet. The quantities in the last column are obtained by reducing to mm the Hartmann T for each instrument. The important effect of irradiation in building up the 40-inch image is at once apparent. In the case of the reflector it should be noted that the star from which the scale is made is practically on the axis. If it were off the axis the theoretical diameter of the cone would be rapidly increased by the aberration of the mirror. With the 4-inch aperture of the doublet the theoretical diffraction disk becomes so large that the performance of the lens brings the diameter of the cone well within the diffraction disk.

EFFICIENCY FROM LIMITING MAGNITUDES

An idea of the comparative efficiency of the three telescopes is best gained from examination of plates taken under conditions varying as they do in actual practice. The most uncertain factor seems to be the state of the atmosphere, its transparency and steadiness, and these are entirely beyond control. Figures 25, 26, and 27 show curves connecting limiting magnitude with the exposure-time, a logarithmic scale of the latter being used. The form of the curve when equal increments of exposure-time are used as abscissae is also shown in Fig. 27. The plotted points inclosed in circles represent the mean of from two to five determinations. The separate points show that atmospheric conditions considered good enough for photography permit a difference of from one to

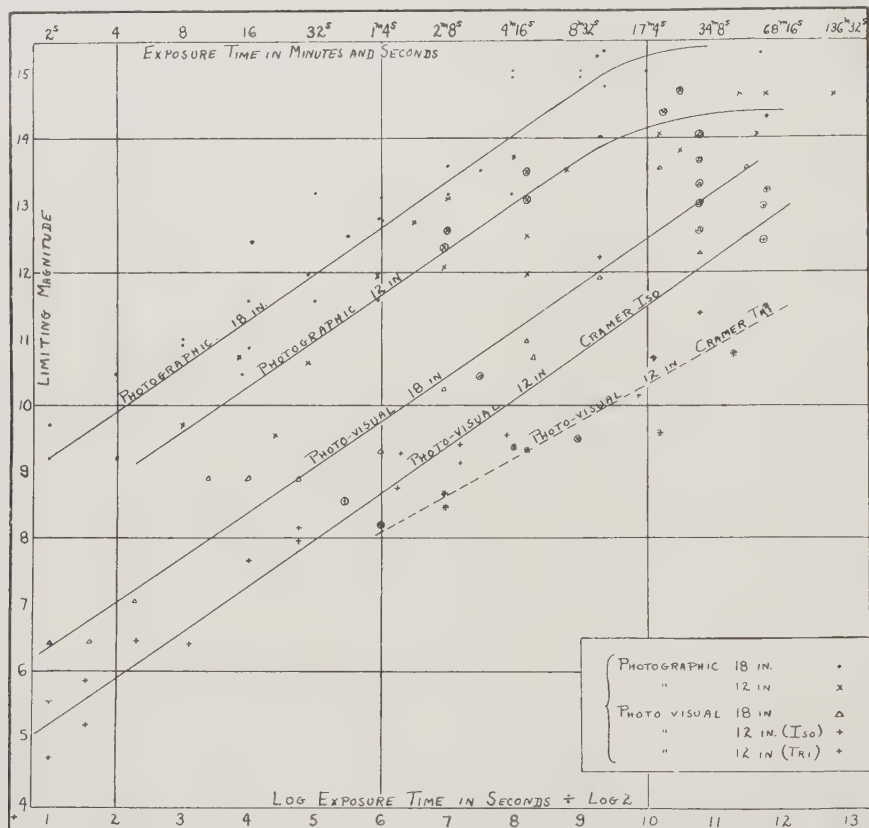


FIG. 25.—Curves of limiting magnitude for reflector

three magnitudes in the limit reached. The fact that the points on the 40-inch curve show less variation is probably due to the fact that the forbidding appearance of the guiding-star often prevents exposures on nights when plates with the doublet and reflector would be taken. Table XXV brings together the outstanding facts shown by the curves. Many of the plates examined for the purpose contained multiple exposures. The limiting magnitudes on photographic and photovisual plates are given for the exposure-times noted at the head of columns 5 to 9. The theoretical limit of vision ($M=9+5 \log$ aperture in inches) is given for purposes of comparison. On the photographic plates, this is reached in about 10 minutes. The performance of 18 inches with the reflector is quite comparable with that of the 40-inch for the photovisual work.

The range of magnitude covered in convenient exposure-time suggests the usefulness of the UV photographic plates for connecting bright standards with stars of intermediate brightness which may in turn be used to connect with faint stars obtained on R plates. The UV photovisual plates

may be used in the same way, though they demand brighter stars for intermediate standards. The R and O photovisual plates are both useful for faint stars and serve as a check on each other.

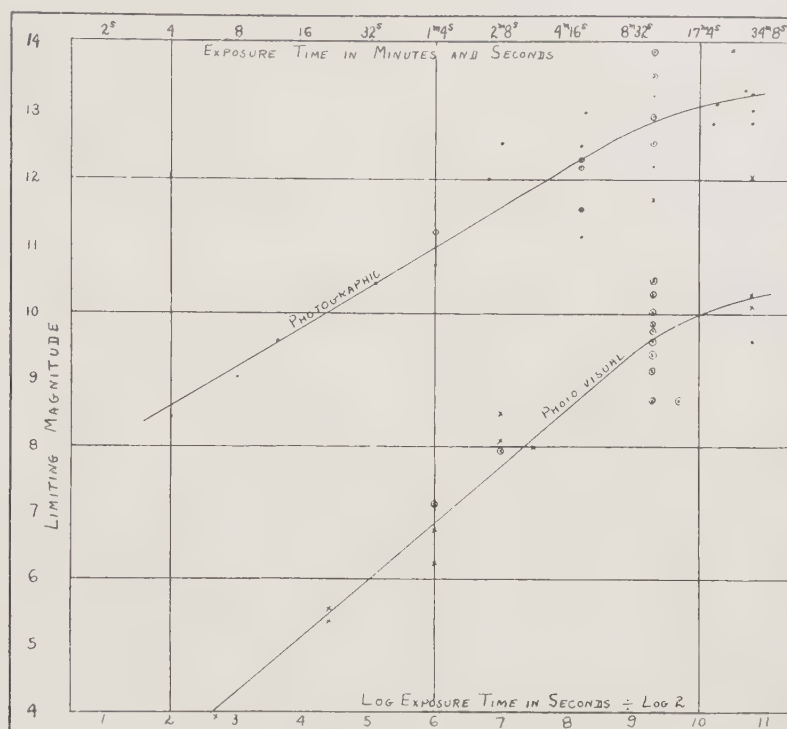


FIG. 26.—Curves of limiting magnitude for UV doublet, 4-inch aperture

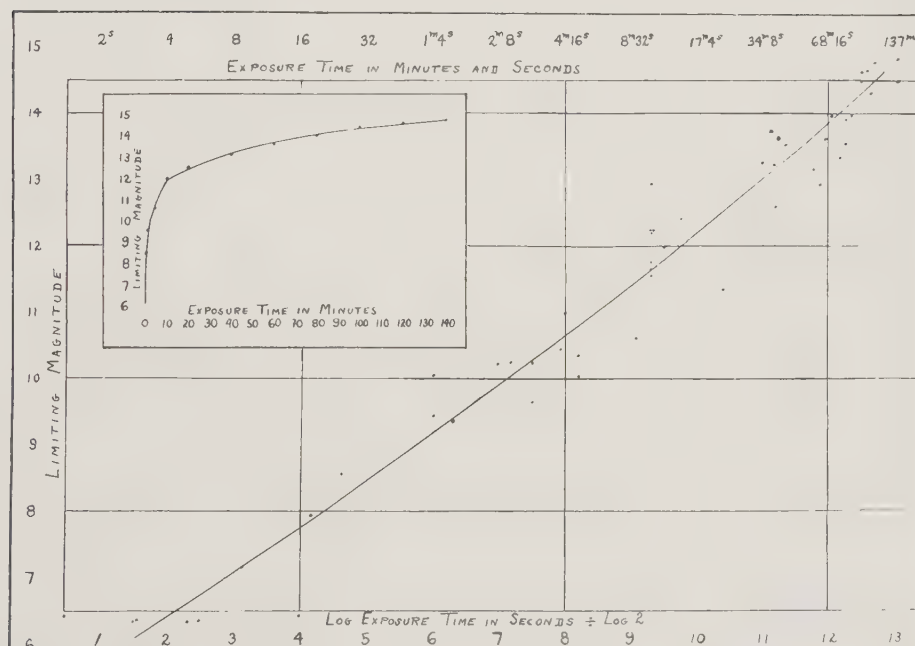


FIG. 27.—Curves of limiting magnitude for 40-inch refractor

TABLE XXV
LIMITING MAGNITUDES FROM PHOTOGRAPHS

SERIES OF PLATES	APERTURE	NO. OF PLATES USED		LIMIT FROM CURVES					THEORETICAL LIMIT OF VISION
		P_h	P_v	10 ^m		30 ^m		60 ^m	
				P_h	P_v	P_h	P_v	P_v	
UV.....	in. 4	25	29	13 ^M	9 ^{M5}	13 ^{M4}	10 ^{M2}		12 ^{M0}
R.....	{12 18	17	20	14.	10.	14.5	11.8	13 ^M	14.4
		9	29	14.8	12.	15.+	13.2		15.3
O.....	40		47		11.5		12.5	13.5	17.0

SUMMARY

1. A description is given of the three telescopes, of the plates used, and of their measurement in the Hartmann microphotometer.

2. The calibration of the field to determine corrections for distance from the optical axis is carried through for each telescope. An attempt is made to correlate the shape of these curves with the optical properties of the system in question.

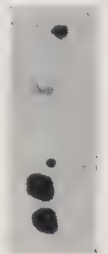
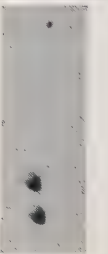
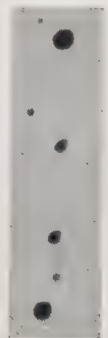
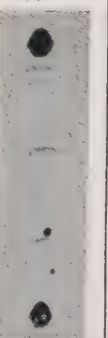
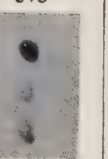
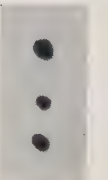
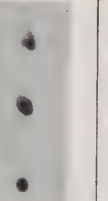
3. The color-curve of the 6-inch doublet is derived from spectra obtained with an objective prism and with a grating. The curve for the 40-inch is reproduced for comparison; the curve for the reflector is a straight line.

4. Curves of zonal foci, astigmatism patterns, and the Hartmann T are derived for the 6-inch and 24-inch and compared with those for the 40-inch determined by Fox.

5. The usefulness of the objective grating for the determination of a scale of magnitudes, with some of its difficulties and limitations, is discussed.

6. A comparison of the actual behavior of the telescopes with theoretical demands is secured through a study of the artificial scales and of the limiting magnitudes obtained with various exposure-times.

PLATE VI

STAR R. No.	568	612	615	DIAMETER OF		EXP. TIME
				APERT.	ZONE	
3771				8	22-24	5
				--	22-24	10
				8	22-24	1
				24	--	1
				22	--	1
3770			471 	22	--	3
				--	20-22	3
				6	20-22	3
				6	20-22	1
				22	--	1
3768			615 	18	--	4
				--	16-18	4
				8	16-18	4
3774				18	--	3
				--	12-16	3
				9	12-16	3
3769				12	--	6
				--	11-12	6
				6	--	6
				6	11-12	6
				6	{ 11-12 16-18	6
3752				24	--	1
				18	--	3
				12	--	10

GROWTH OF IMAGE IN FIELD OF 24-INCH REFLECTOR

II. ON THE ASTROMETRIC FIELD OF THE REFLECTOR

INTRODUCTION

The second part of this paper includes an investigation, of a purely practical nature, of the field of the reflector, in an attempt to throw light on the proper manner of setting on images that are not round. The work may be divided into three parts: (1) a study, by means of photographs taken with different zones of the mirror, of the growth of the image in the reflector field; (2) a search for distortion by comparison of scale-values derived from groups of stars at different distances from the axis, settings on the images being made in various ways; (3) a search for distortion by simple comparison of distances between groups of stars on plates containing exposures centered at three different positions 10' apart.

GROWTH OF IMAGE

The isolation of certain portions of the mirror by means of diaphragms makes it possible to determine what part of the photographic image is formed by each zone of the mirror. These different parts differentiate themselves best, of course, in the case of stars at a considerable distance from the optical axis, but the relative importance of the different zones of the mirror in forming images of stars on the axis, with the evidence of the part played by photographic irradiation in building up the image, is a matter of interest.

Plate VI shows rows of star-images from plates taken under the conditions there specified, where the diameters of the central aperture and of the zones left uncovered are indicated. On R 3774, instead of the whole zone, rectangular openings within it were used. The plate shows images of a bright star 7' from the axis (B.D. $+56^{\circ}568$ of magnitude 6.60), a bright star 40' from the axis (B.D. $+55^{\circ}612$, 6^m05), and in several cases of a fainter star 42' from the axis (B.D. $+55^{\circ}615$, 9^m2). On plate R 3770 the images of B.D. $+56^{\circ}471$, 6^m20, at $r=42'$ in a different direction from the axis, are substituted for those of 615. It should be noted that the flat mirror makes useless a region about 6 inches in diameter in the center of the 24-inch mirror. A detailed study of the original plates reveals much that is of interest. Attention may here be called to a few points:

1. In general, the central zones of the mirror form the dense part of the image nearest the axis, which may be called the nucleus. The greater the radius of the zone the greater the distance from the nucleus of the light it sends to the image.

2. The image formed by the whole aperture out to a certain diameter in a given exposure-time in all cases noticeably exceeds the limit shown by an equal exposure with the outermost zone alone. This indicates the very great part that irradiation plays in building up the image. The law that the whole equals the sum of all its parts does not seem to hold in photographic action, at least in its visible effects.

3. The 22- to 24-inch zone contributes practically nothing of value to an image either on or off the axis.

It appeared, from the foregoing study, that the most consistent way to make settings of a micrometer wire on images of different shapes (because at varying distances from the axis) would be to set in each case on the part of the image formed by a single part of the mirror, for example, on the nucleus formed by the central eight inches. This would mean bisecting central images and setting on the point nearest the axis of distant images. On well-blackened images lying between the two extremes, the position of the nucleus would have to be estimated by reference to fainter images lying near.

DISTORTION FROM SCALE-VALUES

Three plates were measured in the new screw measuring machine¹ and reduced by least squares, with the idea of finding out whether the scale-values determined from groups of stars at various distances from the optical axis would be more consistent from settings on nuclei or from settings which bisect the entire visible image. Table XXVI gives for each plate its number in the R series, the field, the aperture used, the serial number of the solution made, the exposure-time, the distance from the axis and number of stars in each group used, and finally the values with probable errors found for the plate constants p and p' , obtained either by settings on the nuclei or bisections of

TABLE XXVI
DISTORTION FROM SCALE VALUES

PLATE No.	FIELD	APERTURE	SOL. No.	EXP. TIME	DIST. FROM CENTER	No. OF STARS	FROM BISECTIONS				FROM NUCLEI			
							p	$p.e.$	p'	$p.e.$	p	$p.e.$	p'	$p.e.$
R 3691...	χ Cygni	in. 18	III	m 15	8' or less	8	-0.0011	± 3	-0.0002	± 8				
			IV	5	8' or less	8	-.0010	± 4	-.0022	± 6				
			V	15	15' $\pm$	10					-0.0002	± 1	-0.0003	± 1
			VI	5	15 $\pm$	10					-.0005	± 2	-.0004	± 1
			X	15	20-25'	8					+.0011	± 1	+.0004	± 1
			XI	5	20-25'	8					+.0010	± 1	+.0003	± 1
R 3739...	χ Persei	18	XX	10	Central	9	-.0014	± 4	-.0001	± 7				
			XXI	10	12-16'	10					-.0001	± 1	-.0009	± 1
			XXII	10	12-16	10	-.0009	± 1	-.0017	± 1				
			XXIII	10	19-23	8					-.0000	± 1	-.0006	± 1
			XXIV	10	19-23	8	-.0010	± 0	-.0015	± 0				
			XXV	10	Central	9	-.0017	± 2						
			XXVI	10	12-16'	10					-.0012	± 1		
			XXVII	10	12-16	10	-.0020	± 0						
			XXVIII	10	19-23	8					-.0010	± 1		
			XXIX	10	19-23	8	-.0018	± 1						
R 3768...	χ Persei	18 8+zone	I	4	5-6'	6	+.0018	± 4						
			II	4	8-9	7	+.0020	± 2						
			III	4	12-13	6	+.0016	± 2						
			IV	4	15-16	5	+.0018	± 1						
			V	4	17-19	6	+.0024	± 2						
			VI	4	20	5	+.0019	± 1						
			VII	4	23-25	5	+.0019	± 1						

the whole image. The constants p and p' represent the corrections to the assumed scale-value in right ascension and declination. They have a slightly different significance in each case, according to the method of reduction. R 3691 was reduced by means of positions taken from the *Astrographic Catalogue*, so that p represents a correction to the assumption that the ratio of the reflector and astrographic scales $= \frac{300''}{87.38} = 3.4333$. R 3739 contains two identical exposures. The reductions used the mean of the measures of the two exposures which showed reasonably good agreement. The plate was reduced by means of the standard co-ordinates given by Miss Young in *Columbia Contributions No. 24*. These are given in seconds of arc, and the measures on the reflector were reduced with the factor $1 \text{ mm} = 87.38$, so that p in this case represents a correction to this assumed value. This plate was also reduced by the 4-constant method (solutions XXV to XXIX), the measures being corrected beforehand for refraction and aberration, yielding only one scale-constant p . In the reduction of R 3768 (see Plate VI) settings were made on the *nuclei* produced by the central

¹ *Publications of Yerkes Observatory*, 4, Part I, p. 65.

eight inches of the mirror, and reduced to measures which were *bisections* of the images from the full 18-inch aperture, in such a manner that

$$X(18 \text{ in.}) = (1+p)X(8 \text{ in.})$$

So far as can be judged from these three sets of reductions, (1) the scale-value as determined from stars at different distances from the optical axis increases systematically with increasing distance, when the settings are made on the nuclei; (2) the scale-value obtained from bisections is less than that from nuclei, as would be expected; and (3) no systematic change in scale-value appears, when the settings are bisections. For use when a setting on the nucleus is the only one practicable, Table XXVII shows the correction that might apply to the scale-value derived from stars scattered

TABLE XXVII
DISTORTION FROM SCALE VALUES

Aperture	Dist. of Star from Axis		Correction to Mean Scale Value		Possible Correction to Position of Star
	in.	mm.	mm.		
18	15'	10.3	-0.0003	-0".026	0".27
18	22	15.1	-0.0007	-0.061	0.92

over the field if it were to be applied to a star at the distance from the axis given in this table, with the possible corresponding correction to the position of the star. These quantities were derived from 18-inch plates. The quantities of Table XXVI define no such corrections in cases where the shape of the image is such that bisection of the image seems reasonable, as it does to a distance of from 20' to 25' in the 18-inch field.

DISTORTION FROM DISTANCES

In this case, the work was planned with a view to determining what difference appears in the length of a certain fixed interval on the plate, when that interval moves from a position on the axis to other positions off the axis. For this purpose three duplicate exposures, centered 10' apart, were made on a single plate with various apertures. The second and third were differentiated from the first by a short exposure close by, preceding the longer exposure in one case, following it in the other. The interval measured was in most cases the distance between the mean position of several stars in each of two groups. Each plate offered several such intervals, each one occupying three successive mean positions on the plate, approximately 10' apart. The groups were chosen as nearly as possible in line with the direction of displacement of the telescope between exposures, this being in right ascension for all the plates except the last three in Table XXVIII, for which the displacement was in declination. Table XXVIII gives an index of the plates used. The first four columns are self-explanatory. The fifth and sixth give the number of groups and the total number of stars measured on each exposure. The mean hour angle, zenith distance, and temperature follow.

Measures in X , direct and reversed, were made with the screw machine, and approximate Y 's were read from the vertical millimeter scale for use in applying corrections for refraction. The distances from the central star (on the first exposure) or from the computed center (on the second and third) were corrected for refraction and aberration (the latter corrections were neglected after the first group of plates) by the use of an expression of the form

$$X' = X + (M_x + A)X + (N_x - B \tan \delta)Y$$

where X' is the corrected co-ordinate, X and Y the measured co-ordinates, M_x and M_y refraction

coefficients computed from the formulae given by Jacoby,¹ A and B the aberration coefficients as defined by Zuerhellen,² and δ the declination of the plate center. The refraction constant β together with the coefficients for refraction and aberration are given for each plate in the last five columns of Table XXVIII.

TABLE XXVIII
INDEX OF PLATES WITH DUPLICATE EXPOSURES

PLATE NO.	FIELD	APER- TURE	EXP. TIME	NO. OF GROUPS	NO. OF STARS	MEAN H.A.	MEAN Z.D.	TEMP. C.	β	UNIT=0.000001			
										M_z	N_z	A	B
R 3745	χ Persei	18	5	7	32	+4 48	47°	+ 4.7	58.2	+582	-361	-64	-71
3746	χ Persei	18	6	7	32	+5 36	52	+ 3.7	58.4	+696	-343	-64	-71
3992	χ Cygni	18	5	4	11	-0 20	11	+17.7					
3993	χ Cygni	24	3	4	11	-0 8	10	+17.3	55.6	+270	- 3		
3994	χ Cygni	12	7	4	11	+0 39	13	+17.0					
4032	χ Cygni	24	6	4	11	+1 21	18	+10.5	56.9	+296	- 32		
4046	Y Cassiopeiae	12	10	10	15	+3 11	33	- 2.0	59.9	+413			
4049	Y Cassiopeiae	24	5	10	15	+3 25	35	- 0.7	59.8	+433			
4050	Y Cassiopeiae	18	7	10	15	+4 7	41	- 0.9	59.9	+504			
4056	Y Cassiopeiae	24	5	10	15	+1 1	17	- 7.7	61.3	+309			
4057	B.D. +53°52	24	5	10	10	+1 29	19	- 7.9	61.4	-134	+305		
4058	B.D. +53°52	12	10	10	10	+2 54	31	- 8.4	61.5	-246	+298		
4059	B.D. +53°52	18	7	10	10	+3 50	38	- 8.8	61.5	-300	+301		

All the plates were measured all the way through by means of settings which were bisections, and in some cases settings were also made on the nuclei. The mean distance from the center (in X) of the several stars in each group was tabulated for each exposure, as a basis for determining the intervals. These distances are shown in Table XXIX for the bisections on plate R 3745, in order to make clear the effect of the successive exposures on the various groups.

TABLE XXIX
CORRECTED CO-ORDINATES ON PLATE R 3745

Group No.	Exp. 1	Exp. 2	Exp. 3
I.....	mm - 0.4257	mm + 6.3883	mm +13.1002
II.....	- 7.0985	- 0.2933	+ 6.4245
III.....	-14.5331	- 7.7273	- 1.0102
IV.....	-21.2943	-14.4741	- 7.7485
IVa.....	-29.0487	-22.2294	-15.4951
V.....	+ 6.5150	+13.3236	+20.0431
VI.....	+14.0553	+20.8721	+27.5905

Six intervals were formed from these seven groups and the length of each interval in three different positions was determined. Table XXX carries the reduction farther, giving the pair of groups comprising each interval, and for each exposure the position of the groups referred to the optical axis, the mean r for the interval, and the length of the interval (D). The letters p and f mean "preceding" and "following" the center respectively.

For each plate, points were plotted with co-ordinates mean r and change in D (ΔD), the value of D for the smallest mean r being taken as zero. Thus the six intervals of Table XXX yield six groups of three points each, whose co-ordinates are shown in Table XXXI.

¹ *Astronomical Journal*, 17, 17, 1896.

² *Darlegung und Kritik der zur Reduktion photographischer Himmelsaufnahmen aufgestellten Formeln und Methoden*, pp. 50 and 70.

The points were plotted in such a way as to differentiate them according to length of interval used as a basis for the ΔD 's, and also according to size of image, where the grouping had taken this factor into account. Systematic differences from these causes, however, did not appear.

A noteworthy characteristic of the separate curves so obtained is that the curves for a given field, taken with various apertures, even if secured at a different hour angle and on different dates, bear a much more decided resemblance to each other than plates of different fields, taken with the same aperture. The curves from the plates centered on the star B.D. $+53^{\circ}52'$, where the displacement between exposures was in declination, are rather more consistent than the others, perhaps due to the fact that uncertainties from guiding error enter chiefly in the direction of right ascension.

TABLE XXX
POSITIONS AND INTERVALS ON PLATE R 3745

Groups	Exp. 1			Exp. 2			Exp. 3		
	Position	Mean r	D	Position	Mean r	D	Position	Mean r	D
II-I.....	10'-1'p	6'p	6.6728 ^{mm}	0'-9'f	5'f	6.6816 ^{mm}	11'-20'f	16'f	6.6757 ^{mm}
III-II.....	21-10 p	16 p	7.4346	11-0 p	6 p	7.4340	0-11 f	6 f	7.4347
IV-III.....	31-21 p	26 p	6.7612	21-11 p	16 p	6.7468	10-0 p	5 p	6.7383
IVa-IV.....	42-31 p	36 p	7.7544	32-21 p	26 p	7.7553	21-10 p	15 p	7.7466
I-V.....	0-11 f	6 f	6.9407	9-20 f	15 f	6.9353	20-31 f	25 f	6.9429
V-VI.....	11-21 f	16 f	7.5403	20-30 f	25 f	7.5485	31-41 f	36 f	7.5474

TABLE XXXI
 ΔD ON PLATE R 3745
Unit=0.0001 mm

Mean r	ΔD	Mean r	ΔD	Mean r	ΔD
5'f.....	0	5'p.....	0	6'f.....	0
6 p.....	-88	16 p.....	+85	15 f.....	-54
16 f.....	-59	26 p.....	+229	25 f.....	+22
6 f.....	0	15 p.....	0	16 f.....	0
6 p.....	-7	26 p.....	+87	25 f.....	+82
16 p.....	-1	36 p.....	+78	36 f.....	+71

A mean curve was plotted for each plate and these are shown, grouped according to aperture, in Figure 28. A mean curve for each aperture is shown, the means from the separate plates being weighted according to the number of points from which they were originally derived.

As far as they go, the 18-inch curves confirm the results obtained from the scale-values of the previous section—namely, that effects of distortion out to a distance of 15' are not evident when the settings are bisections. On the 12-inch plates the ΔD 's are so small as to be negligible out to a distance of 20'. On the 24-inch plates the measured interval shows beyond 15' a decided tendency to decrease, as if the tendency in bisecting distant, arrow-shaped images was to give too little weight to the fan-shaped portion which falls off in intensity. For two 18-inch plates (R 3745 and R 3746) and one 24-inch plate (R 4056) points are plotted derived from settings on the nucleus. These confirm the results indicated in the previous section qualitatively, in that they show the measured interval decreasing, i.e., the scale-value increasing with distance from the axis.¹

¹ Results obtained more recently from reflector plates (Harriet McWilliams Parsons: "Astrometric and Photometric Statistics of Certain of Hagen's Fields Photographed with the 24-inch Reflector," Doctor's Thesis, University of Chicago, 1921) overlap the present work in two details only. Miss Parsons finds that a scale-value of 87"21 to the mm fits the system of the *Astrographic Catalogue* more closely than the assumed value 87"38. The scale-values in Table XXVI, derived from one plate

The mean value of the 113 intervals which were employed on the 13 plates is 9.8 mm. Table XXXII is an attempt to express numerically the indications of the curves of Figure 28. The values of ΔD are not read from the mean curves, since the residuals for the separate curves are in some

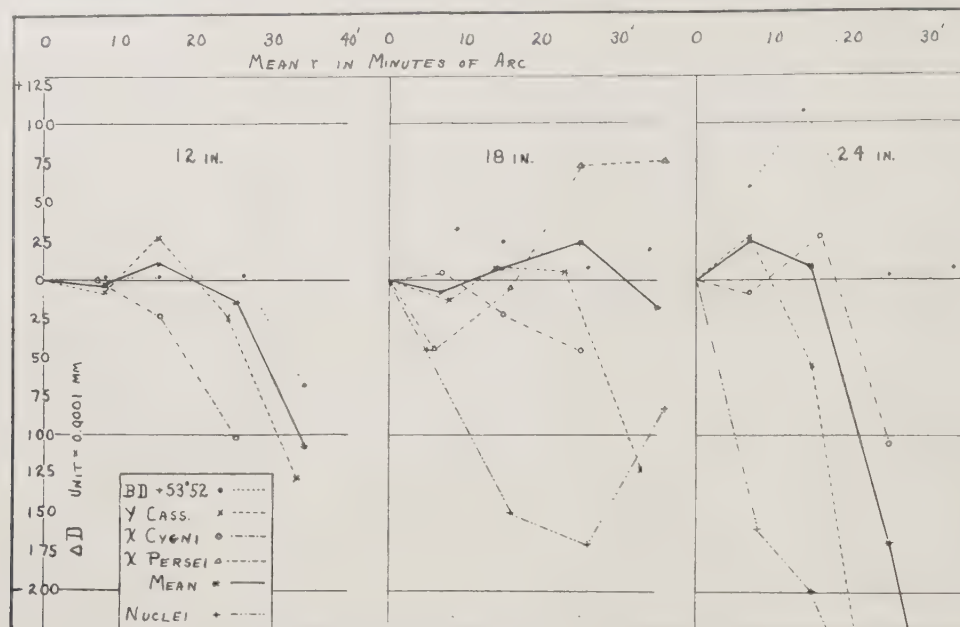


FIG. 28.—Distortion from distances in field of reflector

TABLE XXXII
DISTORTION FROM DISTANCES

Aperture	Dist. from Axis		ΔD	Correction to Scale Value		Possible Error in Position
		mm	mm			
12-inch	15'	10.3	± 0.0025	± 0.0003	± 0.023	0.24
	25	17.2	$- .0050$	$- .0005$	$- .045$	0.77
	35	24.0	$- .0110$	$- .0011$	$- .098$	2.35
18-inch	15	10.3	$\pm .0025$	$\pm .0003$	$\pm .023$	0.24
	25	17.2	$\pm .0050$	$\pm .0005$	$\pm .045$	0.77
	35	24.0	$\pm .0075$	$\pm .0008$	$\pm .067$	1.61
24-inch	15	10.3	$\pm .0035$	$\pm .0004$	$\pm .031$	0.32
	25	17.2	$- .0170$	$- .0017$	$- .151$	2.60
	35	24.0	$- 0.0330$	$- 0.0034$	$- 0.294$	7.06

cases large, but represent less conservative estimates of the possible errors found. The rest of the data is analogous to that given in Table XXVII.

reduced with astrographic positions and one with Miss Young's results for χ Persei (*Columbia Contributions No. 24*), indicate a value near $87''.25$.

For her 24-inch plates Miss Parsons finds a definitely smaller scale-value than for the 18-inch plates. On plate R 3768 (see Table XXVI) images obtained with 8-inch aperture were reduced to those obtained with 18 inches. The consistently positive values of p indicate a smaller scale-value for 18-inch aperture than for 8 inches. These were obtained from settings which were bisections, whereas Miss Parsons states that her tendency in measuring was "to strike a mean between the center of gravity of the image and the center of the nucleus."

So far as they go, both investigations bring out one more point: (1) when fields of different apertures are compared, it appears that both Miss Parsons' settings (between nucleus and center of gravity) and those of the writer (at center of gravity) are too far from the axis to give a uniform scale-value; (2) when images are measured at different distances from the axis in a field of given aperture, settings on the nuclei seem to be too near the axis, and bisections (at center of gravity) appear to give a uniform scale-value. These two indications are contradictory. That is to say, we seem to be on a different basis in comparing different apertures than when comparing different distances with the same aperture.

In conclusion it should be stated that this investigation of the astrometric field of the reflector is avowedly fragmentary and incomplete. The 4-constant solutions in Table XXVI are subject to error in orientation through 90° due to the limited precision in reading position angle from the measuring machine. A kit has already been constructed, tests of which show that the plate clipped to its movable circle will turn through an angle of 90° , accurate to $10''$. Further solutions with the above uncertainty removed should be made in order to check the accuracy of results given here.

SUMMARY

1. A study is made of the growth of the image in the reflector field under the action of various zones of the mirror.
2. In a search for distortion in the reflector field by two different methods, settings on nuclei are compared with settings which are bisections of the whole image. From settings on nuclei, both methods indicate scale-values which increase with distance from the axis; from bisections, no systematic change in scale-value appears out to moderate distances from the axis; beyond, the scale-value increases.
3. Miss Parsons' results permit comparison in a few points.

Grateful acknowledgment of aid in these investigations is here made for appropriations from the Rumford Fund in 1916 and the Watson Fund in 1919, and for fellowships from Mount Holyoke College and the University of Chicago; also to Director E. B. Frost for frequent advice as well as for the use of the equipment of the Yerkes Observatory, and to Professor J. A. Parkhurst for help, of both a practical and inspirational nature.

August, 1920

